Mobile Robot Map-Based Localization using approximate locations and the Extended Kalman Filter *

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Abstract

Map-Based Localization approaches use a local map of the sensed environment that is matched against a previously stored map to correct the robot localization in the world. In many cases these methods are based on a probabilistic representation of the spatial uncertainty and use the Kalman Filter (KF) or the Extended Kalman Filter (EKF) to update the robot's location estimation. On the other hand, Fuzzy Logic has been widely used to generate robust and efficient navigational behaviors for mobile robots in spite of the presence of noise and non-linearities in the system. In this paper we introduce a map-based localization approach that combine a fuzzy robot's location, a possibilistic method to propagate the uncertainty in the robot's motion and the use of the EKF to decrease the spatial uncertainty when valid landmarks are found. Experiments in simulation and in the real world are shown to validate the proposal.

Keywords: Robotics, Map-Based Localization, Approximate Locations, Kalman Filter

*This work has been supported by the MCYT under Project TIC2003-04900

1 Introduction

The localization method shown in this work is included within the map-based localization approaches [3]. Thus the idea of the localization method is, first, perceptual information is gathered by the robot and some kind of local map is built. The uncertainty and vagueness in the sensor data and in the robot's location have to be taken into account. Next, the local map must be matched to a previous map using some matching procedure. Finally, from the results of the matching between both objects of both maps the robot's location is corrected.

In these approaches the Kalman Filter or the Extended Kalman Filter [7] have been widely used as good tools to diminish the uncertainty in the robot's location when the matching between the maps has been obtained. On the other hand, the uses of Fuzzy Logic in robotic systems, to generate robust and efficient navigational and perceptual behaviors, in spite of the presence of noise and non-linearities in the system, have been numerous [9].

In this work we are interested in dealing with the re-usability of existing probabilistic knowledge about the Kalman Filter and to manage a possibilistic approach to represent the robot's location and the propagation of the uncertainty in the robot's motion. Thus we use an approximate representation of the robot's location by means of fuzzy sets, a possibilistic method to propagate the uncertainty and when the conditions of the environment are appropriate the Extended Kalman Filter is applied to diminish the uncertainty in the robot's location. In this way, on one hand, we use fuzzy techniques to represent the robot's location and its motion since these techniques have proved to be effective in controlling systems that are significantly non-linear, which may operate under conditions of great variability. On the other hand, we also use the existing background about the probabilistic techniques by means of the iterative Extended Kalman Filter to correct the robot's location when the conditions of the environment are appropriate.

Now we described the contents of the proposal. First, Section 2 shows the model of the robot's motion, then in Section 3 the possibilistic techniques used to represent the approximate robot's location, the propagation of the uncertainty and the map matching process are explained. In Section 4 we describe how the EKF, a probabilistic technique, is used to estimate the robot's location. In Section 5 the overall process is summarized and Section 6 shows some experimental examples of the robot localization. Finally, Section 7 shows some conclusions and questions for future works.

2 The model of the robot's motion

The model of the robot's motion is summarized in Figure 1. Taking into account the Figure 1, the forward movement of the robot is given by the vector I_k that links two consecutive locations of the robot. This vector

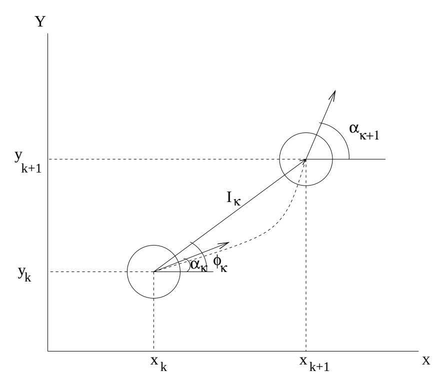

Figure 1: Robot motion model

forms an angle ϕ_k with the X axis. In addition the robot changes its own orientation in $\Delta\alpha_k = \alpha_{k+1} - \alpha_k$. The values of the robot's location $\{x, y, \alpha\}$ in the step k and in the step $k+1$ are known thanks to the odometric system but we are interested in studying the relationship between both locations to consider the influence of the odometric errors. Thus, we suppose a relationship between both locations defined by the following equations:

$$x_{k+1} = x_k + I_k \cdot \cos\phi_k;$$

$$y_{k+1} = y_k + I_k \cdot \sin\phi_k;$$

$$\alpha_{k+1} = \alpha_k + \Delta\alpha_k.$$

To consider the errors of odometric system, the experiment of square bidirectional trajectories of Michigan University (UMBmark) [2] has been carried out in our robot. This experimental test gives us a measure of the odometric errors. Using this measure, we have applied a percentage of the error to consider the values of I_k and $\Delta\alpha_k$, that we call γ_I and γ_α respectively. We suppose that the values of error follow a normal probability distribution so that the error ω_I in I_k is defined by $\omega_I \sim N(0, \epsilon_I)$ with $\epsilon_I = \gamma_I \cdot I_k$, and the error ω_α in $\Delta\alpha_k$ is defined by $\omega_\alpha \sim N(0, \epsilon_\alpha)$ with $\epsilon_\alpha = \gamma_\alpha \cdot \Delta\alpha_k$.

3 Possibilistic techniques

3.1 Approximate robot's location

Our approach needs to know the initial robot's location in the environment from a coordinates frame, although this information can be approximated. From this information, and taking into account a model of the error in the robot's motion, a region in which the robot can be located is obtained.

To consider the uncertainty in the robot's location, we use the approach described in [10] where the concept of approximate location is defined by a fuzzy subset of a given space, read under a possibilistic interpretation. That is, the approximate robot's location is represented by a fuzzy set Pos_k, then the membership function $\mu_{Pos_k}(x, y)$ indicates the degree of possibility that the robot be located at (x, y) in the step k. Actually, we do not work with this fuzzy set directly but we work with the projections of Pos_k on the X, Y axis. Let be $ProX(Pos_k)$ the projection on X of Pos_k and $ProY(Pos_k)$ the projection on Y of Pos_k, they are defined by fuzzy sets with triangular membership functions represented by three values $\{x_0, x_1, x_2\}$ with x_1 the mode and x_0, x_2 the points that define the support of the fuzzy set.

Likewise, the orientation of the robot is measured by the degrees of the angle formed by the forward direction of robot with the X axis and it is represented by a new triangular fuzzy set α_{kf}.

At the beginning of the navigation these fuzzy sets are initialized with the initial robot's location including certain level of uncertainty. When the robot moves around the environment the uncertainty will be propagated taking into account the model of the robot's motion described in Section 2 and the model of uncertainty propagation that we describe below.

3.2 The propagation of the uncertainty

Once we have a model of the robot's motion and of the odometric error related to the translation and rotation of the robot, we apply the approach on Spatial Reasoning shown in [4] to compute the approximate robot's location after it moves from an initial known location.

The errors related to translation and rotation are used to define two symmetrical triangular fuzzy sets. Let $I_{kf} = \{I_k - \omega_I, I_k, I_k + \omega_I\}$ and $\phi_{kf} = \{\phi_k - \omega_\alpha, \phi_k, \phi_k + \omega_\alpha\}$ are two fuzzy sets, they represent the fuzzyfication of the forward vector I_k and the angle ϕ_k, respectively. Figure 2 shows the fuzzy region (and its projections) where the robot can be located after a translation I_{kf} with angle ϕ_{kf}. From the fuzzy region where the robot arrives, $ProX(Pos_{k+1})$ and $ProY(Pos_{k+1})$ can be computed by means of the following process. Let F_1 be a fuzzy set that represents the possibility distribution of the location B in relation to the location A. Let F_2 be a fuzzy set that represents the possibility distribution of the location C in relation to the location B. That is: $\Pi_{A,B} = F_1$ and $\Pi_{B,C} = F_2$. Then, the question is how to determine the possibility distribution of C in relation to A. To solve it, the compositional inference rule [11] can be used: $\Pi_{A,C} = F_1 \circ F_2$. The arithmetic of fuzzy numbers is used to compute $\Pi_{A,C}$ since the fuzzy sets involved are symmetrical fuzzy numbers. Thus, the composition operation is solved using the sum operation of two fuzzy numbers [8]. Then: $\Pi_{A,C} = F_1 \oplus F_2$ where $\oplus$ is the sum operator of two fuzzy numbers.

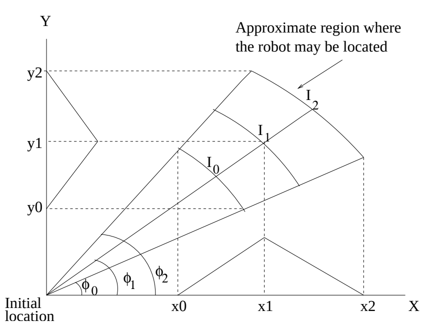

Figure 2: Projections of the fuzzy region of the forward movement I_{kf} with angle ϕ_{kf}

Thus, let $ProX(Pos_k)$ be a fuzzy set that represents the projection on X of the approximate robot's location in the step k. Let $ProX(I_{fk})$ be the fuzzy set projection on X of the forward advance I_{fk} of the robot from the location Pos_k. Then, the projection on X of the new location $ProX(Pos_{k+1})$ is: $ProX(Pos_{k+1}) = ProX(Pos_k) \oplus ProX(I_{fk})$.

To update the projection on Y a similar process is followed. To update the orientation of the robot α_{fk+1}, the fuzzy set $\Delta\alpha_{fk} = \{\Delta\alpha_k - \omega_\alpha, \Delta\alpha_k, \Delta\alpha_k + \omega_\alpha\}$ is considered. This fuzzy set $\Delta\alpha_{kf}$ is summed to the fuzzy set that represents the current robot orientation α_{kf} using the $\oplus$ operator.

3.3 The map matching process

To carry out the robot localization, a local map built during the robot navigation, is matched to a previous one built in a previous exploration phase. Both maps are built using the approach proposed in [1]. This approach let us to build fuzzy segment maps from the ultrasound sensor data and the exploration of the environment, in a way similar to the approach proposed in [5]. The map matching process is carried out by matching of the segments of both maps. For it, we use a matching level among fuzzy segments so that we will be able to identify a fuzzy segment of the local map with the fuzzy segment of the previous map with highest matching level.

The matching operator is defined taking into account two aspects. First, both segments have to be co-linear segments and second, their projections on X and Y must have an high level of overlapping. Using the matching level, the segment of the stored map with the highest matching level to the local segment, is taken into account to correct the robot's location by means of the Extended Kalman Filter as we explain in the following section.

4 Probabilistic techniques

4.1 The EKF applied to the localization problem

The Kalman Filter is a probabilistic method widely used to achieve an optimal estimation of the state of a system from its iterative performance. The filter uses a model of the performance of the system and a model of the measures to compute an expectation on the measures that is matched to the real sensed measures. Both the model of the system and the model of the measures can be affected by noise that is considered as a normal probability distribution. The filter receives as input information the measures sensed by the system and it produces as output information, on one hand, the innovation that is the difference between the prediction and the observation, and on the other hand, the estimate state of the system after considering the correction supplied by the innovation. The Extended Kalman Filter is used instead of Kalman Filter when the equations that define the dynamic of the system, or the equations that define the computation of the measures, or even both of them, are non-lineal. Thus, the Extended Kalman Filter let us to fuse several noisy sources of information obtaining a good, from the statistic point of view, estimation of the state of the system that is consistent with the information from the noisy sources.

In the case of the robot localization in our proposal, these sources of information are the odometric data as coordinates and its orientation, that is $\{x, y, \alpha\}$ and the distance to certain walls of the environment. Using both sources of information the robot's location can be corrected in spite of the presence of uncertainty on both cases. For that, the distance to the walls computed from the approximate estimation of the robot's location is matched against the distance to the walls sensed by the sensory system of the robot. To carry out this process it is needed, on one hand, to define the model of the robot's motion. Thanks to that model we will be able to predict the state of the system. On the other hand, it is needed also to define the way to obtain the real measures and how to compute the prediction of these measures taking into account the prediction of the state of the system. Then we also need a model of the measures. Both models are shown below.

4.2 The model of the system

The model to represent the dynamic of the system has been shown in Section 2. To represent the effects of the odometric errors we rewrite the third equation of the robot's motion in the following terms: $\alpha_{k+1} = \alpha_k + \beta_k + \phi_k$ being $\beta_k = \Delta\alpha_k - \phi_k$. Thus, the evolution of the robot's motion taking into account the errors in the values of ϕ and I is represented by:

$$\begin{bmatrix} x_{k+1} \\ y_{k+1} \\ \alpha_{k+1} \\ \phi_{k+1} \\ I_{k+1} \end{bmatrix} = \begin{bmatrix} x_k + I_k \cdot \cos\phi_k \\ y_k + I_k \cdot \sin\phi_k \\ \alpha_k + \phi_k \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \beta_k \\ \Delta\phi_k \\ \Delta I_k \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ v_\phi \\ v_I \end{bmatrix}$$

that is interpreted as

$$x(k+1) = f(x(k)) + u(k) + v(k), \quad v(k) \sim N(0, Q(k))$$

where $f(x(k))$ is a non-lineal function of the system state, $u(k)$ is the vector of the control input to lead the system to the step $k+1$ from the step k and $v(k)$ is a vector of noise $N(0, Q(k))$.

4.3 The model of the measures

The way to compute the prediction of the measures is using the distance from the robot's center to the walls situated perpendicular to any ultrasound sensor. Let (x_k, y_k) be the center of the robot and representing the sensed wall by means of the equations of straight lines in the way of cartesian normal equation, that is $r_i \equiv e_i x + f_i y + g_i = 0$, then the distance of the robot center to r_i is computed by:

$$d(k)_i = |e_i x_k + f_i y_k + g_i|.$$

To group the distances to several walls we use the following matrix to represent the model of the measures $H(k)$:

$$H(k)x_k = \begin{bmatrix} |e_1 x_k + f_1 y_k + g_1| \\ |e_2 x_k + f_2 y_k + g_2| \\ . \\ . \\ . \\ |e_m x_k + f_m y_k + g_m| \end{bmatrix}.$$

In order to take into account the presence of noise in the sensory system, the final model of the measures is computed by:

$$z(k) = H(k)x(k) + w(k), w(k) \sim N(0, R(k))$$

where $R(k)$ represents the covariance of the noise in the measures. Note that the filter works taking into account measures obtained from different walls and that the number of sensed walls can change while the iterative performance of the filter. This fact let us a great flexibility in the robot navigation. The filter is able to fuse the information from the different sensors with the current prediction on the robot's location to obtain a good estimation on the real robot's location and diminishing its uncertainty.

4.4 The EKF equations of the system

Because the equation system related to F is a non-lineal system, it is needed to use the equations of the Extend Kalman Filter. Thus, F is approximated by the Jacobian matrix of F that is $f_x(k)$ being applied to the estimation of the state of the system $\hat{x}(k|k)$. Then, $f_x(k) = \left[\frac{\partial F}{\partial x}\right]_{x=\hat{x}(k|k)} =$

$$\begin{bmatrix} 1 & 0 & 0 & -I_k \sin\phi_k & \cos\phi_k \\ 0 & 1 & 0 & I_k \cos\phi_k & \sin\phi_k \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}_{x=\hat{x}(k|k)}.$$

In relation to the model of the measures we directly use the matrix H defined above. Therefore, the final equations to use are shown below.

First, the predictions of the state and the covariance are computed by:

$$\hat{x}(k+1|k) = f(\hat{x}(k|k), u(k))$$

$$P(k+1|k) = f_x(k)P(k|k)f_x^T(k) + Q(k).$$

Once we have the measures of the distance to the walls, the innovation can be computed by:

$$\nu(k+1) = z(k+1) - H(k+1)(\hat{x}(k+1|k))$$

and the covariance of the innovation by:

$$S(k+1) = H(k+1)P(k+1|k)H^T(k+1) + R(k+1)$$

after that, the gain of the filter can be computed by:

$$W(k+1) = P(k+1|k)H^T(k+1)S^{-1}(k+1)$$

and the estimation of the state of the system and the covariance of the state are finally obtained by:

$$\hat{x}(k+1|k+1) = \hat{x}(k+1|k) + W(k+1)\nu(k+1)$$

$$P(k+1|k+1) = P(k+1|k) - W(k+1)S(k+1)W^T(k+1).$$

5 The localization process

Once the different elements that are involved in the robot localization have been explained, in this section the overall localization process is described. First the robot begins its navigation from an initial location that is expressed using the concept of approximate location of Section 3.1 and this approximate location is updated using the method explained in Section 3.2. While the robot navigates it is building a local map of the environment that is matched to the previous stored map by means of the method of Section 3.3. If the process of matching let us to identify the walls with an high level of certainty then these walls can be used to correct the robot's location using the EKF as we saw in Section 4.4. One problem arises when the sensed walls can not be identified by the process of matching. In this case the robot stops and tries to find out the best perceptual state to self-localization using the walls situated near to distinguished places like corners, corridors or hallways so that its location can be corrected. In the worst case, if the robot becomes completely lost, then our approach should be adapted to deal with this situation. One solution could be to maintain several hypothesis on the robot location until the environment exploration lets to the robot to know the actual location. This solution has been applied in [10] obtaining good results.

However a main aspect of our proposal has not been solved yet. It is the relationship between the possibilistic method to represent the robot's location and the probabilistic method to correct the robot's location. The transformations that we have carried out in our proposal are based on the work of Gupta [6] that proposes a method to transform possibilistic information into probabilistic information for investment decisions. In that paper, Gupta defines a suitable possibility/probability consistency principle called Budgeting Possibility/Probability Consistency Principle (BPPCP) and also, an associated measure of the consistence. The method suggested converts fuzzy data into normal probability distribution and it can be better applied in those cases where Symmetrical Fuzzy Numbers (SFNs) are used (it is our case). Expressing a SFN as $(x_1 \pm s)$, it is symmetrical around x_1 where $\mu_x(x_1) = 1$ and s is known as spread. Thus, from the information of SFN about a variable, the mean and the variance of related probability distribution can be determined from x_1 and s respectively. The required probability distribution for SFN may be taken as a normal probability distribution with parameters $Mean = x_1$ and $SD = s/z$ where z is an arbitrary variate to be determined so as to ensure that

$$\int_{-\infty}^{+\infty} p(x)dx \approx \int_{x_1-s}^{x_1+s} p(x)dx \approx 1.$$

The normal probability distribution obtained satisfies the BPPCP and provides the maximum level of the associated measure of consistence. Gupta suggests to the variable z a value equal to 4 in the example shown in the paper, so that we have also used $z = 4$.

Using this approach, first it is needed to transform the information expressed in a possibilistic way to a probabilistic form in order to set the initial values of the EKF, second place the robot's location is corrected using the EKF and the measures of the distance to the walls and finally the probabilistic information about the robot's location is transformed to a possibilistic representation coherent with the transformation method explained above. Below this three steps are described.

To use the EKF it is needed to supply to the filter certain input information as the initial values of the robot's location $\{x, y, \alpha\}$ and the covariance matrix of the state of the system. The terms of the diagonal of the covariance matrix represent the variances related to the variables that are present in the state vector and express the underlaying uncertainty. The uncertainty under the probabilistic approach is supposed that follows normal probability distributions. Then, we need to determine the values of the mean and the standard deviation of such as distributions. For achieve that, the information contained into the fuzzy sets is taken into account to use the transformation method of Gupta. We recall that our robot is able to keep certain values about the approximate robot's location using the method explained in Section 3. The idea is to use the values of support and mode of each fuzzy set $(\{x_0, x_1, x_2\})$, to set the initial values of $\{x, y, \alpha\}$ and their variances. In each variable, the mode of the fuzzy set x_1 is taken to set the mean value m of the normal distribution $N(m, \sigma)$. In relation to the standard deviation σ, it is computed taking as value of 4 for z. Thus $\sigma = (x_1 - x_0)/4$. Applying these rules the values needed to set the initial values of $\{x, y, \alpha\}$ and the covariance matrix of the system are obtained.

In relation to the values of the noise present in the system $Q(k)$ the same approach to compute the odometric errors of Section 2 has been used to keep the coherence of the error model. To model the noise presents in the measures of the ultrasound sensors, for each sensor s_i, the noise is considered that follows a normal distribution $\omega_{R_i} \sim N(0, R_i)$. For each sensor, R_i is computed depending on the distance of the sensed wall and the orientation of that wall in relation to the direction of the beam of the ultrasound sensor. Thus, the measures affected by a lesser error are walls near to the sensor and perpendicular to the direction of the beam. The use of several values for the error of the measures is important in order to let the filter to give more weight to the best measures.

When the filter has been properly initialized, it carries out its performance estimating the robot's location and using the information of the measures to correct the estimation of the robot's location. When the robot location has been corrected, then the probabilistic information is used to generate an approximate location to obtain again the possibilistic model. We follow the inverse process. For example, in the case of the variable x, a fuzzy set $\{x_0, x_1, x_2\}$ is generated with x_1 the mean of the normal distribution for x and $x_0 = x - 4\sigma$ and $x_2 = x + 4\sigma$ being σ the standard deviation associated to x.

6 Experimental Results

The proposal has been implemented in a mobile robot Nomad 200 and it has been validated first in simulation and afterwards in an office-like environment of the real world. Figure 3 shows an example in a simulated environment. In this example, the robot follows the contour of a wall situated on its left side and corrects its location several times while it follows the wall. The robot's motion is

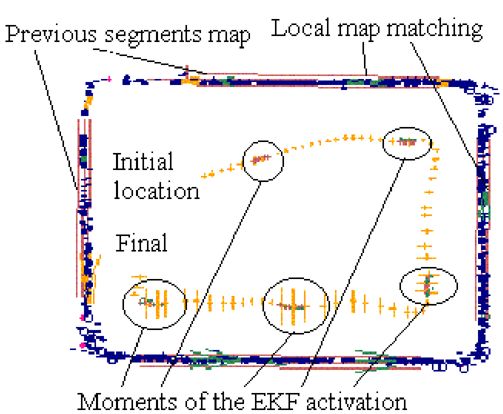

Figure 3: First localization example

represented by the little crosses. The crosses become bigger if the uncertainty associated to the robot's location increases. The uncertainty increases faster when the robot has to turn to follow the wall since the rotations are the motions that incorporate more uncertainty to the system. The stored segments form part of a fuzzy segments map built in a previous exploration phase. While the robot navigates around the environment it builds a local fuzzy segment map. The robot's location is activated when certain level of uncertainty is achieved. Then, the robot applies the map matching process to find valid sensory references and to be able to apply the EKF. The moments of the activation of the EKF are depicted by circles, so we show that in a few iterations the robot's location is corrected and its uncertainty is reduced.

The second example, shown by Figure 4, is carried out in the real world. In this exper-

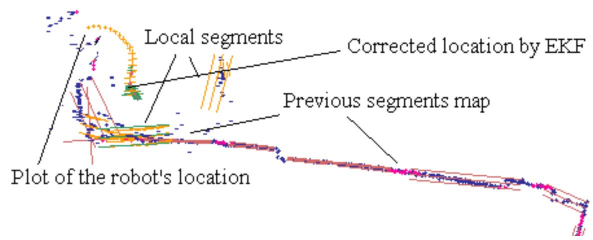

Figure 4: Second localization example

iment the robot begins its navigation from the left of the environment. When the uncertainty increases to certain level then the robot tries to correct its location. Again, the robot uses the local segments map which are represented by three parallel lines situated on the sensed points on the left, on the right and at front of the robot's trajectory. Using the explained matching process, the robot can identify the sensed walls and uses the EKF to correct its location. In this example, the robot corrects its location and then it stops.

7 Conclusions and future work

In this work we use possibilistic techniques to represent the robot's location, the propagation of the uncertainty on the robot's motion and to carry out the map matching process. On the other hand the EKF, a probabilistic technique, is used to correct the robot location when valid references are found as a result of the map matching process. There are several reasons to use of different uncertainty representation models in the tasks involved in our proposal. First, in our behavior-based architecture the behaviors that connect perception to action are designed by fuzzy rules and use concepts of fuzzy control in their performances. Also the perception model to establish the contexts of application of the different behaviors uses fuzzy rules and fuzzy concepts. Thus, we need to maintain a fuzzy model to represent the robot location to be consistent with other levels of our architecture. Second, fuzzy techniques offer a good performance under non-linear conditions and under the presence of noise in the system and in the sensor data. Third, fuzzy methods for localization exist, as for example [10], but we think that in [10] the assurance of the localization method depends highly on the kind of landmark that the robot is able to find using the sensory system. This generates high dependence on the correct detection of the landmarks. On the contrary, by using the segment maps to carry out the matching, the robot is able to correct its location more frequently. Moreover, the high background probabilistic that exists in the area of robot localization is used to correct efficiently the robot's location by means of the EKF. Also, in this work we show an example of the use of both possibilistic and probabilistic techniques in the same system. Among the advantages of Kalman Filter we found the following ones. It uses all available information that it gets to make an overall best estimate of a state, being optimal under certain assumptions. It is recursive, then not all data needs to be kept in storage and re-processed every time. Finally, it is a data processing algorithm or filter, then it tries to obtain an optimal estimate of variables from data coming a noisy environment.

Regarding the future work, several aspects of the proposal need further research. The extension of the proposal when the initial robot's location is unknown or the robot becomes completely lost and the study of other approaches to carry out the possibilistic/probabilistic transformation. Also, we will study the incorporation of some method to resolve the simultaneous localization and map building problem.

References

[1] E. Aguirre and A. González. Integrating fuzzy topological maps and fuzzy geometric maps for behavior-based robots. *International Journal of Intelligent Systems*, 17(3):333–368, 2002.

[2] J. Borenstein and L. Feng. UMBmark: A benchmark test for measuring dead reckoning errors in mobile robots. In *SPIE Conference on Mobile Robots*, 1995.

[3] J.L. Crowley and P. Reignier. World modeling and position estimation for a mobile robot using ultrasonic ranging. In *Proc. of the IEEE Int. Conf. on Robotics and Automation*, pages 674–680, 1989.

[4] S. Dutta. Approximate spatial reasoning: Integrating qualitative and quantitative constraints. *International Journal of Approximate Reasoning*, 5:307–331, 1991.

[5] J. Gasós and A. Rosseti. Uncertainty representation for mobile robots: Perception, modeling and navigation in unknown environments. *Fuzzy Sets and Systems*, 107:1–24, 1999.

[6] C. P. Gupta. A note on the transformation of possibilistic information into probabilistic information for investment decisions. *Fuzzy Sets and Systems*, 56:175–182, 1993.

[7] L.D. Hostetler and R.D. Andreas. Nonlinear kalman filtering techniques for terrain-aided navigation. *IEEE Transactions on Automatic Control*, 28(3):315–323, 1983.

[8] A. Kaufmann and M. Gupta. *Introduction to Fuzzy Arithmetic: Theory and Applications*. Van Nostrand Reinhold, 1985.

[9] A. Saffiotti. The uses of fuzzy logic in autonomous robot navigation. *Soft Computing*, 1:180–197, 1997.

[10] A. Saffiotti and L.P. Wesley. Perception-based self-localization using fuzzy locations. In M. van Lambalgen, editor, *Reasoning with Uncertainty in Robotics. LNAI*, pages 368–385. Springer, 1996.

[11] H.J. Zimmerman. *Fuzzy Set Theory and Its Applications*. Kluwer, 1985.