Fuzzy Linguistic Multicriteria Morphological Analysis in Scenario Planning

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***Abstract*—Companies that want to be competitive must make use of good practices to anticipate the future by analyzing the possible effects of today's decisions on their own long-term development. Scenario planning is among the most extended approaches to accomplish this. One of the techniques often used in scenario planning is Morphological Analysis, which aims to explore the space of feasible futures in a systematic way by analyzing all the combinations of the possible states of the variable that compose the system under study. Each of this combinations represent a possible future scenario. This work focuses on a particular variant known as the MORPHOL method, in which every scenario is evaluated in terms of the probability it eventually arises. This is computed using the marginal probability estimates of the hypothetical variables' states involved in the scenario, which are given by human experts. This method presents two drawbacks: first, the probabilities have to be expressed by numerical values which makes difficult its estimation by humans and does not capture its uncertainty; and second, it examines the scenarios basing only on their probability, thus it may ignore scenarios that are interesting but not the very probable. In order to ease the experts' task and capture their opinions in a better way, we introduce Computing with Words techniques. For solving the second shortcoming, we apply Multi-criteria Decision Making to uncover good scenarios according to several criteria jointly including probability. The result is a novel linguistic multi-criteria method for morphological analysis that has been successfully applied to a real problem and thus deserves further research.**

I. Introduction

Nowadays, business markets are in constant change. Companies competing for a place in them are aware of the need of specific tools to face complex decisions that may have a great impact in the future, especially those concerning innovation. The use of good practices is a must in order to anticipate the future by analyzing how their present decisions can affect their own long-term development. One of the most widely employed techniques for this purpose is Scenario planning [1], [2], [3], which consists of a set of tools that emphasize the systematization of creative thinking to uncover possible future scenarios.

Although the definition varies depending of the author, a scenario can be defined as a future situation together with the steps that lead from the current state to it. The main strength of scenario planning methodologies is their systematic basis, assisted by computerized tools which take into account possibilities that may have escaped to ad-hoc human studies and highlight implications and impact of present decisions.

Many proposals have been presented in the Scenario planning literature [3]. One of them is Morphological Analysis (MA) [4], originally conceived as a more general problem solving technique that has been successfully applied to the scenario planning field. The key aspect of MA is the systematic generation of all possible combinations of values that the variables involved in a problem can take, so that every possibility is taken into account, even those that are unfeasible due to physical, logical, social or material reasons, which are later discarded.

The present contribution focuses on the MORPHOL method [5], an adaptation of MA proposed by M. Godet as part of a general toolbox[1] in which morphological analysis is one of the stages of a prospective study. The particularity of MORPHOL is the evaluation of every combination in terms of the probability of its constituents, in order to obtain a general picture of what the future may look like. This evaluation is based on the subjective opinions of a panel of experts. Hence the result is vague and imprecise – something inherent to the human language and judgments. However such vagueness is not properly addressed by MORPHOL since the experts have to express the probabilities as crisp numbers.

To overcome these issues and ease the task of the experts to evaluate the alternatives in a more natural way and according to more criteria apart from probability, we propose the use of soft-computing techniques such as Computing with Words (CW) and linguistic Multi-Criteria Decision Making (MCDM) into the morphological analysis stage. This work presents a novel procedure for morphological analysis in scenario planning that incorporates these elements. It is intended as a continuation of the research on scenario planning tools carried by the authors [6].

The contribution is structured as follows. Morphological analysis and the MORPHOL method are reviewed in Section II. Our model is presented in Section III together with the fundamentals of the techniques employed, mainly linguistic labels and MCDM. Section IV contains a sample problem where we apply our method together with a discussion of the results. Section V is devoted to conclusions and further work.

[1]The software tools can be found at http://en.laprospective.fr /methods-of-prospective/downloading-the-applications.html

II. Morphological Analysis

General Morphological Analysis [4] was first proposed as a technique to tackle complex problems with non-quantifiable variables. Despite being originally applied to a number of problems of different nature, mainly modular system design [7], in the last decade Ritchey [8], [9], [10], [11] used MA in scenario planning. The steps of the method are listed below (to better illustrate them we used the example of a simplistic car design problem):

1) Identify the variables of interest in the problem. *Car design problem: Gear, Engine and Energy Source.*
2) Identify the possible (categorical, non-numeric) values each variable can take: *Gear*=$\{$*Manual (M), Automatic (A)*$\}$*, Engine*=$\{$*Combustion (C), Electric (E), Hybrid (H)*$\}$ *and Energy Source*=$\{$*Gasoline (G), Diesel fuel (D), Electricity (L)*$\}$.
3) Identify inconsistencies between values of different variables, and summarize them in a cross-consistency matrix. For instance, *an electric engine is not compatible with gasoline or diesel fuel as energy source.*
4) Generate all feasible combinations or alternative solutions for the problem, in a combinatorial manner. A solution is a situation in which each variable takes one value in its corresponding domain, e.g. *(Manual, Combustion, Diesel fuel), (Automatic, Hybrid, Diesel fuel), etc.*

Steps 1 and 2 are very important from the point of view of the success of the method, but no specific formal procedure is indicated for them. This knowledge is usually extracted from domain experts, who also provide a cross-consistency matrix (step 3) containing a degree of consistency or compatibility for every pair of values between two different variables. The cross-consistency matrix of our car design example may look like Table I. A consistency degree of 1 indicates an incompatible pair of values, for instance *Electricity* as Energy Source and *Combustion* as Engine type: a totally electric car cannot have a combustion engine. This means that combinations of the form (·, Combustion, Electricity) cannot arise (are unfeasible), no matter the value of the Gear ("·" represents an indifference), because variables *Engine* and *Energy source* cannot take the values *Combustion* and *Electricity* within the same combination. A consistency degree of 2 means partial compatibility, and 3 is total compatibility. Although three different compatibility degrees have been employed in the example, any other scale is also possible.

As conceived by Ritchey, MA is a computerized interactive decision support tool. After providing the cross-consistency matrix to the program, the user fixes the values of several variables involved in the problem, and then launches a combinatorial analysis that generates and displays all the feasible combinations of the remaining variables, where feasibility is expressed in terms of mutual consistency of the values taken by all the variables. This way, it provides an overview of all the situations that may happen because they are compatible with the values fixed by the user.

TABLE I
Example of a cross-consistency matrix in the car design problem with three different consistency grades 1,2,3

		Gear		Engine		
		M	A	C	E	H
Engine	C	3	3			
	E	2	3			
	H	2	3			
Energy source	G	3	3	3	1	2
	D	3	3	3	1	3
	L	3	3	1	3	3

The method does not evaluate the generated combinations using numerical values apart from the internal consistency of the values involved in each alternative.

A. MA with MORPHOL

As aforementioned, MORPHOL can be seen as an adaptation of MA to scenario planning. In MORPHOL terminology, every value that a variable can take is called an *hypothesis*. For instance, in an economic study, a variable could be "most frequent type of business in the city", and its possible hypotheses would be $\{$Traditional familiar businesses, Technological mid-sized businesses, Large multinationals$\}$. The program does not make use of a cross-consistency matrix but the user can manually introduce the groups of incompatible hypotheses of two or more variables, which will be used to discard unfeasible combinations (scenarios), i.e. those containing some of the forbidden groups indicated.

Before generating the scenarios, the user can also specify the probability that a variable takes each of its hypotheses, i.e. he/she provides a probability distribution over the hypotheses of the variable. Then, the overall probability that the scenario eventually arises is the joint probability that every variable takes the corresponding hypothesis of the scenario. Assuming that the variables are all mutually independent, this is computed as the product of the marginal probabilities of occurrence of every hypothesis involved. Turning to our example, a feasible scenario, say (Manual, Hybrid, Electricity) has a joint probability of:

$$p(Gear = M, Engine = H, Energy\ source = L) =$$
$$p(Gear = M) \cdot p(Engine = H) \cdot p(Energy\ source = L)$$

These marginal probabilities must be provided by the user.

The output of the system is a list with all feasible scenarios and their joint probability.

Beyond the validity of the variable independence assumption, which is clearly unrealistic in most real problems, some issues arise when following this approach. They are pointed in the next section.

III. Fuzzy Linguistic Multi-criteria Morphological Analysis

A. Motivation

As stated in [12], experts are unwilling to give precise numerical estimates of their opinions or thoughts, and even more when the result should be a probability distribution. Such numbers are usually inaccurate and do not capture the inherent uncertainty of their estimates, thus it is more reasonable to allow them providing their valuations in natural language. In this sense, the use of a linguistic variable *Probability* whose values are linguistic terms may be helpful, for two reasons: firstly, it eases the experts' task in evaluating the probability of every hypothesis by allowing them to just give linguistic assessments, such as *unlikely, very likely, it may*, etc; and secondly, it can capture the uncertainty associated to the subjective character of these terms.

Another aspect that could be improved in MA is the way a scenario is evaluated. MORPHOL only uses the probability criterion, but many other are possible. In addition, it would also be desirable an overall measure that takes into account several criteria simultaneously, so that scenarios can be compared in a more reliable and robust way. This could be accomplished by introducing MCDM models that deal with distinct evaluation criteria. Moreover, such techniques also allow for the aggregation of the opinions of several experts, which was not possible in conventional MORPHOL or MA. However, at this preliminary stage of research we will restrict to only one expert .

In what follows, a new MA proposal is presented to cope with these shortcomings.

B. Linguistic Probabilities

In our approach, the probability of a hypothesis is a linguistic variable. In order to capture and handle the vagueness of linguistically expressed probabilities, we consider more suited a computational model based on membership functions of the labels [13], [14]. We employ trapezoidal fuzzy numbers (TrFN) represented as four-tuples of real numbers, $\tilde{A} = (a, b, c, d), a \leq b \leq c \leq d$ as the mathematical structure that enables computations with linguistic labels. For the linguistic probabilities we employ the same values suggested in [12], which were elicited after psychological studies to determine which ranges of probability are generally considered by humans as *most likely*, *meaningful chance*, etc. (Fig. 1). Note some terms carry more uncertainty than others.

It must be noticed that the introduction of the linguistic variable *Probability* poses specific difficulties not found in other linguistic variables, since the constraint of being a well-defined probability distribution must still be met. To properly deal with this, we follow the approach suggested in [15]. Assume we define a linguistic probability distribution over the possible hypotheses a variable of our problem can take. Let I be the set of these values. Then for each $i \in I$ we use a linguistic term to describe the probability that the variable eventually takes such value. Let π_i be the fuzzy number underlying the linguistic probability label assigned to value i. In order for the linguistic probability to be well-defined [15], it must hold that the sum of all the fuzzy numbers associated to the labels contains[2] the singleton fuzzy number 1_χ, defined as $\mu_{1_\chi}(x) = 1$ if $x = 1$, and 0 otherwise. Formally,

[2] $\tilde{A} \supseteq \tilde{B} \leftrightarrow \mu_{\tilde{A}}(x) \geq \mu_{\tilde{B}}(x) \forall x \in \mathbb{R}$

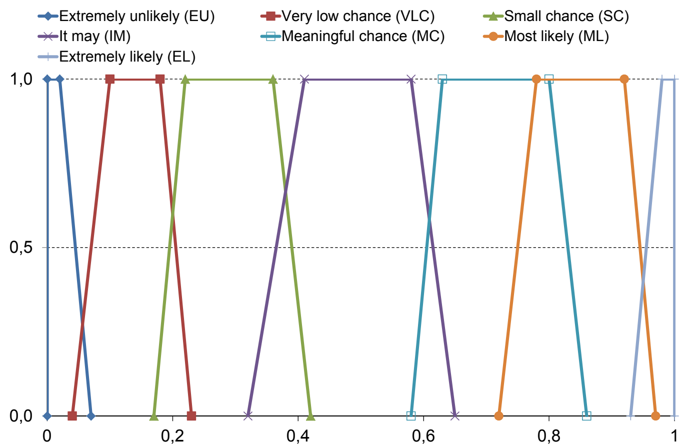

Fig. 1. Membership functions of the possible labels for the linguistic variable probability

$\sum_{i \in I} \pi_i \supseteq 1_\chi$. This condition should be checked when the experts evaluate the probability of the hypotheses of a variable using linguistic labels.

According to [15], Zadeh's extension principle [13] should be applied to operate with the underlying fuzzy numbers of probability labels. Although this is simple for the addition of TrFNs, the product and division do not yield another TrFN and hence the usual approximation has been taken, i.e. computing the 0-cut and the 1-cut and approximating a TrFN with them. Operations with TrFNs have been defined as follows. Let $\tilde{A} = (a_1, a_2, a_3, a_4)$, $\tilde{B} = (b_1, b_2, b_3, b_4)$ be two TrFNs, then

- $\tilde{A} \oplus \tilde{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4)$.
- $\tilde{A} \otimes \tilde{B} = (a_1 \cdot b_1, a_2 \cdot b_2, a_3 \cdot b_3, a_4 \cdot b_4)$.
- $\tilde{A} \oslash \tilde{B} = (a_1/b_4, a_2/b_3, a_3/b_2, a_4/b_1)$.
- $\alpha \cdot \tilde{A} = (\alpha \cdot a_1, \alpha \cdot a_2, \alpha \cdot a_3, \alpha \cdot a_4), \quad \alpha \in \mathbb{R}$.
- Defuzzification: defuzz$(\tilde{A}) = (a_1 + 3a_2 + 3a_3 + a_4)/6$.
- Distance: $d(\tilde{A}, \tilde{B}) = (|a_1 - b_1| + 3|a_2 - b_2| + 3|a_3 - b_3| + |a_4 - b_4|)/6$.

To compute the overall linguistic probability of a scenario, the TrFNs underlying the linguistic probability labels assigned to its constituent hypotheses are multiplied using the fuzzy multiplication operator $\otimes$. If the problem involves n variables and we have assigned linguistic probabilities to the hypotheses that constitute the scenario we want to evaluate, whose underlying TrFNs are $(\tilde{A}_1, ..., \tilde{A}_n)$, then the linguistic joint probability of the scenario is $\tilde{P} = \otimes_{i=1}^{n} \tilde{A}_i = (\prod_i a_i, \prod_i b_i, \prod_i c_i, \prod_i d_i)$.

C. Multiple Criteria in Scenario Evaluation

Although probability is an important criterion, there exist other criteria that may help us uncover interesting scenarios despite not being among the most probable. We suggest three evaluation criteria in addition to probability:

1) Compatibility: although totally unfeasible scenarios are discarded, the remaining ones should also be evaluated according to partial or total compatibility. A scenario that is partially compatible will most likely be less feasible, and thus less relevant, than other with total compatibility.

2) Desirability: most desirable scenarios are those that we should try to reach, as they represent situations that are very favorable for our interests [16].

3) Catastrophicity: this criterion is helpful to indicate dangerous scenarios that should be avoided since involve serious risks. This criterion is not opposite to desirability, since while a scenario could be undesirable, this may not represent a threat for our interests [16].

For analogous reasons as those explained for the probability, we model these criteria as linguistic variables to allow the experts expressing their viewpoints in a linguistic manner. Since they are not probabilities, they can be evaluated using any suitable term set, possibly different from Fig. 1. We propose the next three linguistic labels [9] to describe the compatibility degree: {*Not compatible(N), Partially compatible(P), Totally compatible(T)*}; and the next five for desirability and catastrophicity: {*Very low(VL), Low(L), Moderate(M), High(H), Very high(VH)*}. The underlying TrFNs are given below. They are actually Triangular FNs as the two central values match:

Compatibility:	Desirability and catastrophicity:	
- N = (0,0,0,1);	- VL = (1,1,1,2);	- H = (3,4,4,5);
- P = (0,1,1,2);	- L = (1,2,2,3);	- VH = (4,5,5,5);
- T = (1,2,2,2);	- M = (2,3,3,4);	

In a problem with n variables, the aggregated fuzzy desirability and catastrophicity of a scenario s are calculated as:

$$Desir_s = \frac{1}{n}\big(\oplus_{j=1}^{n}\tilde{D}esir_s^j\big); \quad Catast_s = \frac{1}{n}\big(\oplus_{j=1}^{n}\tilde{C}atast_s^j\big);$$

where $\tilde{D}esir_s^j$ and $\tilde{C}atast_s^j$ are the desirability and catastrophicity TrFNs of the hypothesis taken by variable j in s. The aggregated fuzzy compatibility is computed as

$$Compat_s = \sqrt[r]{\big(\otimes_{i<j}\,\tilde{C}ompat_s^{ij}\big)}$$

where $\tilde{C}ompat_s^{ij}$ is the compatibility of the hypotheses taken by the variables i and j, and $r = \binom{n}{2}$ indicates the number of mutual hypothesis compatibilities that must be checked in a scenario. The aggregated compatibility value is also used to discard scenarios in which this value matches $(0, 0, 0, \cdot)$. In that case the scenario is discarded since it means one or more of the mutual compatibilities involved were *Not compatible*.

D. Fuzzy TOPSIS for Scenario Ranking

The introduction of several criteria requires a MCDM method to rank the scenarios according to multiple criteria simultaneously. MCDM tools enable the user to assign different weights to the criteria in order to fit his/her particular views about their relative importance within the problem. They also allow for sensitivity studies. In this sense, our method uses a linguistic variant of the TOPSIS method (Technique for Order Preference by Similarity to Ideal Solution) [17] to rank the scenarios according to the criteria considered. Although MCDM had been suggested before for morphological design [7], unlike our method, it was not a linguistic proposal and did not fit the particularities of scenario planning. First, we introduce two operations employed during the algorithm:

- Normalization: let $\tilde{A}_i = (a_i, b_i, c_i, d_i), i = 1, ..., n$ be a collection of n TrFNs we have to normalize, and let $t = \max_i\{d_i : i = 1, ..., n\}$. Then the normalized TrFNs are computed like $\tilde{A}'_i = (1/t)\cdot \tilde{A}_i = (a_i/t, b_i/t, c_i/t, d_i/t)$.
- Ideal-max and Ideal-min of a collection of TrFNs:
  Imax$(\tilde{A}_1, ..., \tilde{A}_n)$=(max$\{a_i\}$,max$\{b_i\}$,max$\{c_i\}$,max$\{d_i\}$)
  Imin$(\tilde{A}_1, ..., \tilde{A}_n)$=(min$\{a_i\}$, min$\{b_i\}$, min$\{c_i\}$, min$\{d_i\}$)

The main steps of TOPSIS are summarized below.

1) Identify the n criteria (we propose $n = 4$ as mentioned above), generate the m alternatives (scenarios) and evaluate them according to each criterion. Let J be the subset of criteria to be maximized, and J' the subset of criteria to be minimized, so $|J| + |J'| = n$.
2) Identify the (linguistic) weights of the criteria $\tilde{w}_1, ..., \tilde{w}_n$ and normalize them, $\tilde{w}'_j = \tilde{w}_j / \sum_{t=1}^{n}\tilde{w}_t$.
3) Discard those scenarios whose constituent hypotheses are incompatible.
4) Normalize the TrFNs of every column separately to obtain the normalized matrix $(\tilde{n}_{ij})_{i=1,...m}^{j=1,...,n}$.
5) Build the weighted normalized matrix $(\tilde{v}_{ij})_{i=1,...m}^{j=1,...,n}$ as $\tilde{v}_{ij} = \tilde{w}_i \otimes \tilde{n}_{ij}$.
6) Determine the ideal positive and ideal negative scenarios, $A^+ = (\tilde{a}_1^+, ..., \tilde{a}_n^+)$ and $A^- = (\tilde{a}_1^-, ..., \tilde{a}_n^-)$. Each component is computed as

$$\tilde{a}_j^+ = \begin{cases} \text{Imax}(\tilde{v}_{1j}, ..., \tilde{v}_{mj}) & \text{if } j \in J \\ \text{Imin}(\tilde{v}_{1j}, ..., \tilde{v}_{mj}) & \text{if } j \in J' \end{cases} \quad (1)$$

$$\tilde{a}_j^- = \begin{cases} \text{Imin}(\tilde{v}_{1j}, ..., \tilde{v}_{mj}) & \text{if } j \in J \\ \text{Imax}(\tilde{v}_{1j}, ..., \tilde{v}_{mj}) & \text{if } j \in J' \end{cases} \quad (2)$$

7) Compute the (crisp) distance between each scenario i and the ideal positive and negative scenarios:

$$d_i^+ = \sum_{j=1}^{n} d(\tilde{v}_{ij}, \tilde{v}_j^+); \qquad d_i^- = \sum_{j=1}^{n} d(\tilde{v}_{ij}, \tilde{v}_j^-) \quad (3)$$

8) Compute the relative proximity index for each scenario, $R_i = \frac{d_i^-}{d_i^+ + d_i^-}$. The closer to 1, the better the scenario.
9) Sort the scenarios decreasingly according to their R_i.

IV. Example of Application

A. Problem Description

In order to illustrate how the method is applied, we have selected the real example studied in [18]. It is aimed at generating global scenarios that may arise taking into account the evolution of some global economic, social and political factors. The six variables involved and the probabilities of their hypotheses are listed in Table II. An asterisk indicates the probability of a dummy (meaningless) hypothesis used by MORPHOL to make sure probabilities add to 1. This dummy hypothesis would represent all the possible states of the variable except those specified in their hypotheses. For details on the semantics of the hypotheses, refer to [18].

TABLE II

CRISP MARGINAL PROBABILITIES GIVEN BY EXPERTS

Variable	H1	H2	H3	H4	H5
V_1 (Demography)	0.45	0.35	0.1	0.1 (*)	
V_2 (Geopolitical context)	0.3	0.3	0.15	0.25 (*)	
V_3 (Global. of economy)	0.3	0.4	0.15	0.15 (*)	
V_4 (European integration)	0.3	0.4	0.1	0.2 (*)	
V_5 (Average GDP in EU)	0.15	0.3	0.1	0.1	0.35(*)
V_6 (Unemployment)	0.15	0.3	0.3	0.15	0.1(*)

TABLE III

LINGUISTIC EVALUATION ACCORDING TO PROBABILITY (TOP OF EACH CELL), DESIRABILITY (CENTER) AND CATASTROPHICITY (BOTTOM)

Variable	H1	H2	H3	H4	H5
V_1 (Demography)	IM VL M	SC L L	VLC H VL	VLC* - -	
V_2 (Geopolitical context)	SC VL M	SC L L	VLC VH VL	SC* - -	
V_3 (Globalization of economy)	SC L M	IM L VL	VLC H VL	VLC* - -	
V_4 (European integration)	SC L L	IM M VL	VLC VH VL	VLC* - -	
V_5 (Average GDP in the UE)	VLC L M	SC M L	VLC H VL	VLC VH VL	SC* - -
V_6 (Unemployment)	VLC VH VL	SC H VL	SC L L	VLC VL H	VLC* - -

For translating crisp probabilities into linguistic ones, we simply select the label of Fig. 1 for which the crisp probability has the highest membership degree. The evaluation according to compatibility, desirability and catastrophicity was done by one of the authors who, playing the role of a domain expert, gave his opinion for every hypothesis separately (Table III). The dummy hypothesis was not evaluated for any variable (except for the probability criterion in order to check the linguistic probability distribution is well formed for each variable) nor used in scenario generation, since it does not make sense to give a value for the compatibility, desirability or catastrophicity of a hypothesis that represents all the possible states of a variable except those already specified. The linguistic cross-consistency matrix is not shown due to space constraints. In a good scenario, probability, desirability and compatibility should be maximized while catastrophicity should be minimized.

B. Experiments and Results

The number of scenarios generated initially was $3 \times 3 \times 3 \times 3 \times 4 \times 4 = 1296$. After computing the aggregated compatibility for every scenario, only 248 scenarios were finally retained. The rest were found unfeasible because their aggregated compatibility TrFN was of the form $(0, 0, 0, \cdot)$. Hence $m = 248$ alternatives were passed to the TOPSIS method, considering $n = 4$ criteria.

Our first aim is to compare our ranking with the one obtained by MORPHOL, to be sure the results make sense.

TABLE IV

RANKING COMPARISON TAKING PROBABILITY AS THE SINGLE CRITERION

Scenario	P/Mean	Rank$_G$	Rank$_L$	Prox. index R_i
1 2 2 2 2 3	12.44	1	1	1
1 1 2 2 2 3	12.44	1	1	1
2 1 2 2 2 3	9.68	3	3	0.625
2 2 2 2 2 3	9.68	3	3	0.625
1 1 1 2 2 3	9.33	5	3	0.625
1 2 1 2 2 3	9.33	5	3	0.625
1 1 2 1 2 3	9.33	5	3	0.625
1 2 2 1 2 3	9.33	5	3	0.625

Table IV shows how scenarios are sorted according to Godet's MORPHOL (Rank$_G$) and to our Linguistic method (Rank$_L$) when a (crisp) weight of 1.0 is assigned to probability and 0 to the other criteria. Godet uses the P/Mean measure, defined as the ratio between the probability of a scenario and the average probability of all scenarios, while we use the proximity index R_i defined previously. Both rankings are very similar, as expected. Further, the table shows a lot of ties in the ranking, specially in our method. Although this could be seen as a drawback of our proposal, it is indeed an advantage as it is indicating that all those scenarios with Rank$_L$ = 3 are indistinguishable in practice, whenever a bit of uncertainty is present in the probabilities. That is the reason why several crisp probabilities very closed to each other were translated to the same linguistic label, which in turn produced this result. In other words, the distinction made by MORPHOL between scenarios with P/Mean = 9.68 and those with 9.33 is not real, since the difference between both values is very small and therefore, in practice, it can be interpreted as *no difference at all* when the experts' marginal probability estimates are affected by uncertainty to a certain extent.

Note, in addition, that our method was not conceived to consider a single criterion isolately but in conjunction with other ones, which makes the alternatives much easier to distinguish since ties in the proximity index are almost impossible in those cases. Moreover, in a real environment, experts are expected to use linguistic labels directly, hence no transformation from numerical to linguistic values has to be done.

Our second aim is to show the usefulness of our method in finding interesting scenarios beyond their probability, and to study the effect of the weights given to criteria. In order to do this, we have tested different sets of weights. Although TOPSIS admits fuzzy weights of a linguistic weight evaluation, crips values (represented as singleton TrFNs) have been used at this (preliminary) stage of research. Set of weights (1) gives a weight of 0.25 to every criterion; set (2) gives a weight of 0.7 to the probability and 0.1 to the rest; and set (3) gives 1 to the probability and 0 to the rest.

The results are summarized in Table V. For each set of weights (1), (2) and (3), the proximity index and the rank of a scenario are shown. Table V(a) is sorted by Rank$^{(1)}$, thus only the best scenarios according to set of weights (1) are shown. Next to each scenario, we also display information of how the scenario would be rated according to sets of weights (2) and (3). This way, we can see that the best alternatives according

TABLE V
RANKING COMPARISON OF TWO CRITERION WEIGHT BALANCES

Scenario	$R_i^{(1)}$	Rank$^{(1)}$	$R_i^{(2)}$	Rank$^{(2)}$	$R_i^{(3)}$	Rank$^{(3)}$
3 3 3 3 4 1	0.811	1	0.380	29	0	248
3 3 3 3 4 2	0.794	2	0.375	34	0.004	243
3 3 3 2 4 1	0.775	3	0.369	37	0.010	242
3 3 3 2 4 2	0.759	4	0.369	36	0.024	222
3 3 3 3 3 2	0.697	5	0.329	58	0.004	243
3 2 3 3 4 1	0.671	6	0.317	68	0.004	243
3 2 2 2 4 1	0.666	7	0.356	45	0.083	153
3 2 2 2 4 2	0.663	8	0.398	25	0.164	88
3 3 3 2 3 2	0.662	9	0.323	64	0.024	222

(a) Best scenarios according to set of weights (1): (0.25/0.25/0.25/0.25)

Scenario	$R_i^{(1)}$	Rank$^{(1)}$	$R_i^{(2)}$	Rank$^{(2)}$	$R_i^{(3)}$	Rank$^{(3)}$
1 2 2 2 2 3	0.58918	25	0.80717	1	1	1
1 1 2 2 2 3	0.54553	36	0.78668	2	1	1
2 2 2 2 2 3	0.50282	55	0.56773	3	0.625	3
1 2 2 1 2 3	0.47555	76	0.55493	4	0.625	3
1 2 1 2 2 3	0.47041	80	0.55252	5	0.625	3
2 1 2 2 2 3	0.45917	90	0.54725	6	0.625	3
1 1 2 1 2 3	0.43190	104	0.53445	7	0.625	3
1 1 1 2 2 3	0.42676	108	0.53204	8	0.625	3
1 2 2 2 4 2	0.54284	37	0.52757	9	0.514	9

(b) Best scenarios according to set of weights (2): (0.7/0.1/0.1/0.1)

to (1) are ranked in a notably worse position in (2) and, more interestingly, they are among the worst if we only consider the effect of the probability alone, as done in (3). The reason is that such scenarios, despite being improbable, are very good in all the other criteria and hence they still get a high proximity index according to set of weights (1).

Table V(b) is sorted by Rank$^{(2)}$, hence probability has much more importance than the other criteria. The ranking obtained in this case is very different from Table V(a), and is now very similar to a strict ordering of the probabilities. Notice, however, that taking into account more criteria allows to break the ties generated when considering probability alone (as done by set of weights (3)). Possibly this weighting scheme is more suitable for our interests, since all probable scenarios will get a good rank, and the information of the other criteria will be used to discriminate among them. In addition, it is unlikely that very improbable scenarios get a good rank, even though they may perform very well in the other criteria, because the weight given to probability is very high and cannot be easily compensated by the rest of criteria.

The behaviour exhibited by other balances of the criteria such as 0.4 for probability and 0.2 for the rest, not shown here due to space constraints, was in between: very probable and very improbable scenarios were intermixed on top positions of the proximity index ranking. This demonstrates that a MCDM approach is able to find scenarios that are interesting beyond their probability, thus deserves further research.

V. Conclusions and Further Work

We have proposed a linguistic method for the morphological analysis stage of scenario planning, extending existing a crisp method with CW and MCDM techniques. Our approach eases the task of the experts when evaluating hypotheses that compose an scenario by allowing them to express their views in linguistic terms, and helps in uncovering scenarios that may be worth considering, despite not being among the most probable. We have employed linguistic probabilities as well as other criteria to judge scenarios, and MCDM to obtain a final ranking. The results confirm that taking into account several criteria can be useful to discover promising scenarios as more information is taken into account in the process.

Further work may include a formal sensitivity analysis regarding both different linguistic weights for the criteria and different degrees of uncertainty in their opinions, as well as the incorporation of judgments from several experts and experimentation with other MCDM techniques.

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