Accuracy Improvements to Find the Balance Interpretability-Accuracy in Linguistic Fuzzy Modeling: An Overview

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Abstract. System modeling with fuzzy rule-based systems (FRBSs), i.e. fuzzy modeling (FM), usually comes with two contradictory requirements in the obtained model: the *interpretability*, capability to express the behavior of the real system in an understandable way, and the *accuracy*, capability to faithfully represent the real system. While linguistic FM (mainly developed by linguistic FRBSs) is focused on the interpretability, precise FM (mainly developed by Takagi-Sugeno-Kang FRBSs) is focused on the accuracy. Since both criteria are of vital importance in system modeling, the balance between them has started to pay attention in the fuzzy community in the last few years.

The chapter analyzes mechanisms to find this balance by improving the accuracy in linguistic FM: deriving the membership functions, improving the fuzzy rule set derivation, or extending the model structure.

1 Introduction

System modeling is the action and effect of approaching to a model, i.e., to a theoretical scheme that simplifies a real system or complex reality with the aim of easing its understanding. Thanks to these models, the real system can be explained, controlled, simulated, predicted, and even improved. The development of *reliable* and *comprehensible* models is the main objective in system modeling. If not so, the model loses its usefulness.

There are at least three different paradigms in system modeling. The most traditional approach is the *white box modeling*, which assumes that a thorough knowledge of the system's nature and a suitable mathematical scheme to represent it are available. As opposed to it, the *black box modeling* [74] is performed entirely from data using no additional a priori knowledge and considering a sufficiently general structure. Whereas the white box modeling has serious difficulties when complex and poorly understood systems are considered, the black box modeling deals with structures and associated parameters that usually do not have any physical significance [2]. Therefore, generally the former approach does not adequately obtain reliable models while the latter one does not adequately obtain comprehensible models.

A third, intermediate approach arises as a combination of the said paradigms, the *grey box modeling* [37], where certain known parts of the system are modeled considering the prior understood and the unknown or less certain parts are identified with black box procedures. With this approach, the mentioned disadvantages are palliated and a better balance between reliability and comprehensibility is attained.

Nowadays, one of the most successful tools to develop grey box models is *fuzzy modeling* (FM) [50], which is an approach used to model a system making use of a descriptive language based on fuzzy logic with fuzzy predicates [75]. FM usually considers model structures (fuzzy systems) in the form of fuzzy rule-based systems (FRBSs) and constructs them by means of different parametric system identification techniques. Fuzzy systems have demonstrated their ability for control [28], modeling [63], or classification [18] in a huge number of applications. The keys for their success and interest are the ability to incorporate human expert knowledge – which is the information mostly provided for many real-world systems and is described by vague and imprecise statements – and the facility to express the behavior of the system with a language easily interpretable by human beings. These interesting advantages allow them to be even used as mechanisms to interpret black box models such as neural networks [16].

As a system modeling discipline, FM is mainly characterized by two features that assess the quality of the obtained fuzzy models:

- *Interpretability* — It refers to the capability of the fuzzy model to express the behavior of the system in a understandable way. This is a subjective property that depends on several factors, mainly the model structure, the number of input variables, the number of fuzzy rules, the number of linguistic terms, and the shape of the fuzzy sets. With the term interpretability we englobe different criteria appeared in the literature such as *compactness*, *completeness*, *consistency*, or *transparency.*
- *Accuracy* — It refers to the capability of the fuzzy model to faithfully represent the modeled system. The closer the model to the system, the higher its accuracy. As closeness we understand the similarity between the responses of the real system and the fuzzy model. This is why the term approximation is also used to express the accuracy, being a fuzzy model a fuzzy function approximation model.

As Zadeh stated in its *Principle of Incompatibility* [87], "*as the complexity of a system increases, our ability to make precise and yet significant statements about its behavior diminishes until a threshold is reached beyond which precision and significance (or relevance) become almost mutually exclusive characteristics.*"

Therefore, to obtain high degrees of interpretability and accuracy is a contradictory purpose and, in practice, one of the two properties prevails over the other one. Depending on what requirement is mainly pursued, the FM field may be divided into two different areas:

- *Linguistic fuzzy modeling (LFM)* — The main objective is to obtain fuzzy models with a good interpretability.
- *Precise fuzzy modeling (PFM)* — The main objective is to obtain fuzzy models with a good accuracy.

The relatively easy design of fuzzy systems, their attractive advantages, and their emergent proliferation have made FM to suffer a deviation from the seminal purpose directed towards exploiting the descriptive power of the concept of a linguistic variable [87,88]. Instead, in the last few years, the prevailing research in FM has focused on increasing the accuracy as much as possible paying little attention to the interpretability of the final model.

Nevertheless, a new tendency in the FM scientific community that looks for a good balance between interpretability and accuracy is increasing in importance [3,13,72,79]. The aim of this chapter is to review some of the recent proposals that attempt to address this issue using mechanisms to improve the accuracy of fuzzy models with a good interpretability.

The chapter is organized as follows. Section 2 analyzes the different existing lines of research related to the improvement of interpretability and accuracy to find a good balance in FM, Sect. 3 introduces the most useful kinds of FRBSs to improve their accuracy, Sect. 4 presents mechanisms to increase the accuracy of linguistic fuzzy models, and, finally, Sect. 5 points out some conclusions.

2 Major Lines of Work

The two main objectives to be addressed in the FM field are *interpretability* and *accuracy*. Of course, the ideal thing would be to satisfy both criteria to a high degree but, since they are contradictory issues, it is generally not possible. In this case, more priority is given to one of them (defined by the problem nature), leaving the other one in the background. Hence, two FM approaches arise depending on the main objective to be considered: LFM (interpretability) and PFM (accuracy).

Regardless of the approach, a common scheme is found in the existing literature to perform the FM:

1. Firstly, the main objective (interpretability or accuracy) is tackled defining a specific model structure to be used, thus setting the FM approach.
2. Then, the modeling components (model structure and/or modeling process) are improved by means of different mechanisms to define the desired ratio interpretability-accuracy.

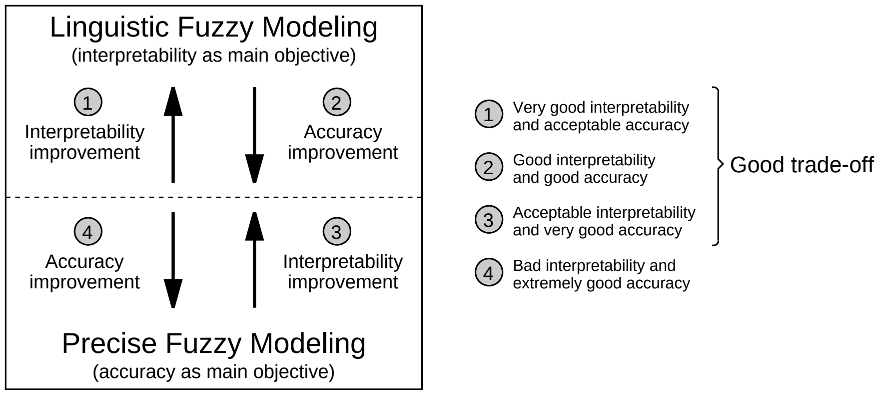

Fig. 1. Improvements of interpretability and accuracy in fuzzy modeling

This procedure results in four different possibilities (see Fig. 1): LFM with improved interpretability, LFM with improved accuracy, PFM with improved interpretability, and PFM with improved accuracy.

Although historically more priority has been given to the accuracy, currently the search of a good balance between both criteria is increasing in importance. Indeed, a significative effort is being performed by several researchers proposing *improvement mechanisms* to compensate for the initial difference. Among the four said lines of work, clearly this philosophy is pursued by two of them: *LFM with improved accuracy* and *PFM with improved interpretability* (approaches 2 and 3 in Fig. 1, respectively).

Moreover, another interesting proposal is *LFM with improved interpretability* (approach 1 in Fig. 1). Although LFM uses a model structure with a high description power by itself, there are some problems (curse of dimensionality, excessive number of input variables or fuzzy rules, garbled fuzzy sets, etc.) that make it not to be as interpretable as desired and the need of interpretability improvements to restore the searched balance is justified.

Finally, the modus operandi of obtaining more accuracy in PFM (approach 4 in Fig. 1) does not pay attention to the comprehensibility of the model and acts close to black box techniques. This approach does not follow the original objective of FM and does not profit from the advantages that distinguish it from other modeling techniques. Although the approach is useful when only accuracy is required, it goes away from the aim of the present book.

This chapter is devoted to review different accuracy improvements that have been proposed to attain the desired balance. Thus, Sect. 4 shows some mechanisms found in the recent literature to do so.

3 Types of Fuzzy Rule-Based Systems

Before presenting the search of a balance interpretability-accuracy in FM by improving the accuracy, it seems that there is need to introduce the different kinds of FRBSs usually employed. It is a significant aspect to consider since depending on the rule structure used, an FRBS has itself a specific capability of description and approximation. The section is only focused on the FRBS types usually considered to improve their accuracy for the sake of a good trade-off, thus having a good description capability themselves.

3.1 Linguistic Fuzzy Rule-Based System

Also known as Mamdani-type FRBS [57,58], the linguistic FRBS constitutes the main tool to develop LFM. A crucial reason why this approach is worth considering is that it may remain verbally interpretable, playing the concept of linguistic variable [88] a central role. Linguistic FRBSs are formed by linguistic rules with the following structure:

IF X_1 is A_1 and ... and X_n is A_n
THEN Y_1 is B_1 and ... and Y_m is B_m ,

with X_i and Y_j being input and output linguistic variables respectively, and with A_i and B_j being linguistic labels with fuzzy sets associated defining their meaning. These linguistic labels will be taken from a global *semantic* defining the set of possible fuzzy sets used for each variable (Fig. 2 shows an example with triangular membership functions). This structure provides a natural framework to include expert knowledge in the form of fuzzy rules.

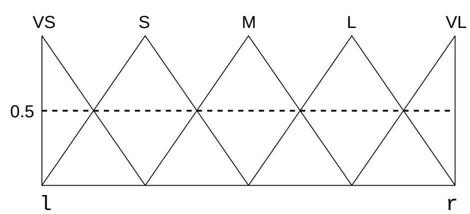

Fig. 2. Graphical representation of an example of the semantic considered for a variable, standing VS for *very small*, S for *small*, M for *medium*, L for *large*, and VL for *very large*, with $[l, r]$ being the corresponding variable domain

In these systems, the knowledge base (KB) – the component of the FRBS that stores the knowledge about the problem being solved – is composed of:

- the *rule base* (RB), constituted by the collection of linguistic rules themselves joined by means of the connective *also*, and

- the *data base* (DB), containing the term sets and the membership functions defining their semantics.

3.2 Singleton Fuzzy Rule-Based System

The singleton FRBS, where the rule consequent takes a single real-valued number, may be considered as a particular case of the linguistic FRBS (the consequent is a fuzzy set where the membership function is one for a specific value and zero for the remaining ones). Its rule structure is the following:

IF X_1 is A_1 and ... and X_n is A_n
THEN Y_1 is y_1 and ... and Y_m is y_m ,

with y_j being real-valued values.

Compared with the linguistic FRBS, the fact of having a different consequent value for each rule (no global semantic is used for the output variable) slightly worsens the interpretability. Nevertheless, the singleton FRBS may be used to develop LFM.

3.3 Fuzzy Rule-Based Classification Systems

A fuzzy rule-based classification system is an automatic classification system that uses fuzzy rules as knowledge representation tool. Therefore, the fuzzy classification rule structure is as follows:

IF X_1 is A_1 and ... and X_n is A_n
THEN Y is C ,

with C being the class label.

Other alternative representations that consider a certainty degree for each rule or that include all the possible class labels with their corresponding certainty degrees in the consequent part are usually also considered.

4 Improving the Accuracy in Linguistic Fuzzy Modeling

LFM has certain inflexibility due to the use of a global semantic that gives a general meaning to the used fuzzy sets. In fact, the use of linguistic variables imposes the following constrains [4,10]:

1. There is a lack of flexibility in the FRBS because of the rigid partitioning of the input and output spaces.
2. When the system input variables are dependent themselves, it is very hard to fuzzy partition the input spaces.

3. The homogeneous partitioning of the input and output spaces when the input-output mapping varies in complexity within the space is inefficient and does not scale to high-dimensional spaces.
4. The size of the KB directly depends on the number of variables and linguistic terms in the system. The derivation of an accurate linguistic FRBS requires a significant granularity amount, i.e., it needs of the creation of new linguistic terms. This granularity increase causes the number of rules to rise significantly, which may make the system lose the capability of being interpretable by human beings. In the most of the cases, it would be possible to obtain an equivalent FRBS having a very lesser number of rules if there would not exist that input space rigid partitioning.

However, it is possible to make some considerations to face this drawback [13]. Basically, two ways of improving the accuracy in LFM can be considered by performing the improvement in

- the *modeling process*, extending the model design to other components different from the RB such as the DB or considering more sophisticated derivations of it, or in
- the *model structure*, slightly changing the rule structure to make it more flexible.

The following three subsections introduce the improvements existing in the literature for designing the DB, learning the RB with sophisticated methods, and extending the model structure.

4.1 Data Base Design

Basic LFM methods are exclusively focused on determining the set of fuzzy rules composing the RB of the model [78,82,85]. In these cases, the DB is usually obtained from expert information (if available) or by a normalization process and it remains fixed during the RB derivation process.

However, the automatic design of the DB has shown to be a very suitable mechanism to increase the approximation capability of the linguistic models. Generally speaking, the procedure involves either defining the most appropriate shapes for the membership functions that give meaning to the fuzzy sets associated to the considered linguistic terms or determining the optimum number of linguistic terms used in the variable fuzzy partitions, i.e., the granularity (e.g., [25,29,68,70]).

In the following, we show different approaches of the DB design regarding the way of integrating it in the derivation process and the effects caused to the membership functions. Some considerations on the interpretability preservation are also discussed.

Integration of the DB design in the whole FRBS derivation process

The design of the DB may be integrated within the whole derivation process of a FRBS with different schemata:

- *Preliminary design* (learning the DB) — It involves extracting the DB a priori by induction from the available data set. This process is usually performed by non supervised clustering techniques [53,64].
- *Embedded design* (learning the KB) — This approach derives the DB using an embedded basic learning method [23,25,26,30,40,68,70]. The technique involves having a simple learning method that designs, from a specific DB, other components of the fuzzy linguistic model (e.g., the RB). Following a meta-learning process, the method generates different DBs and samples its efficacy running the basic learning method.
- *Simultaneous design* (learning the DB together with other components) — The process of designing the DB is jointly developed with the derivation of other components such as the RB in a simultaneous procedure [31] [39,45,48,56,73,81,83].
- *A posteriori design* (tuning the DB) — This approach, usually called DB *tuning*, involves refining the DB from a previous definition once the remaining components have been obtained. It is one of the most common procedures. Usually, the tuning process changes the membership function shapes [7,21,38,46,52,60,76] and the main requirement is to improve the accuracy of the linguistic model. Nevertheless, as shown in the previous section, sometimes another kind of a posteriori DB design is made to improve the interpretability (e.g., merging membership functions [29]).

Of course, several of these approaches can also jointly be considered. For example, in [51] a two-stage DB design is made by first deriving simultaneously the DB and the RB and then performing an a posteriori tuning. In [44], an initial generation of the RB with a subsequent three-stage DB design (input variable selection, simultaneous DB tuning and RB reduction, and DB fine tuning) is developed.

Preliminary, embedded, and a posteriori design approaches are usually combined with other methods to perform the whole derivation process in several sequential stages. Instead, the simultaneous design of the DB together with other components constitutes a whole derivation process itself.

The sequential derivation has the main advantage of reducing the search space since confined spaces are tackled at each stage. On the other hand, in the simultaneous derivation, the strong dependency of the components is properly addressed. However, the process becomes significantly more complex in the latter case because the search space significantly grows making the selection of an appropriate search technique crucial.

Recently, the cooperative coevolutionary paradigm [69] has shown an increasing interest thanks to its high ability to manage with huge search spaces and decomposable problems, and new simultaneous derivation methods are currently emerging using this technique [15,66,67].

Finally, we should say that the DB design gives more flexibility to the modeling process but it runs the risk of losing interpretability and overfitting the problem, wherefore this task must be carefully made. Some mechanisms to keep a good interpretability are discussed later on.

Learning/tuning the membership function shapes

Another interesting aspect to consider when designing the DB is the way of defining the membership function shapes. The most usual approaches are the following:

- *Learning/tuning the membership function parameters* — The most common way of deriving the membership functions is to change their definition parameters [7,21,31,38,39,44–46,48,51,60,67,70,76,81]. For example, if the following triangular-shape membership function is considered:

$$\mu(x) = \begin{cases} \frac{x-a}{b-a}, \text{ if } a \leq x < b \\ \frac{c-x}{c-b}, \text{ if } b \leq x \leq c \\ 0, \quad \text{otherwise} \end{cases},$$

  changing the basic parameters – a, b, and c – will vary the shape of the fuzzy set associated to the membership function (see Fig. 3), thus influencing the FRBS performance. The same yields for other shapes of membership functions (trapezoidal, gaussian, sigmoid, etc.).

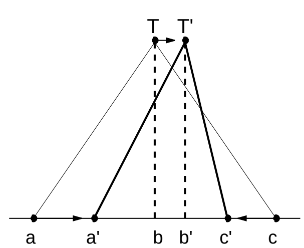

Fig. 3. Definition of the membership function shapes by their parameters

- *Using linguistic modifiers* — Another way to define the membership function shapes of the DB is to use linguistic modifiers [20,52]. Section 4.3 describes them in depth. They involve considering more flexible alternative expressions for the membership functions to vary the compatibility degrees to the fuzzy sets. For example, a new membership function can be obtained raising the membership value to the power of α, i.e.,

$$\mu'(x) = (\mu(x))^{\alpha}, \quad 0 < \alpha.$$

By changing the α value we may define different membership function shapes. Figure 4 shows the effect of this approach.

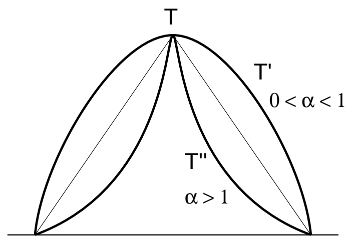

Fig. 4. Definition of the membership function shapes using linguistic modifiers

- *Establishing the context* — A third possibility to change the membership functions is to define the context, i.e, their operation range. It is usually performed by *scaling functions* that map the input and output variables onto the universe of discourse over which the fuzzy sets are defined. From a linguistic point of view, the scaling function can be interpreted as a sort of context information.
  We may distinguish between linear and non-linear scaling functions, which are well known in classical control theory:
  - *Linear context* [34] — A linear scaling function is of the form:

    $$f(x) = \alpha \cdot x + \beta.$$

    The scaling factor α enlarges or reduces the operating range, which in turn decreases or increases the sensitivity of the controller with respect to that input variable, or the corresponding gain in case of an output variable. The parameter β shifts the operating range and plays the role of an offset to the corresponding variable.
  - *Non-linear context* [35,47,55,65] — The main disadvantage of linear scaling is the fixed relative distribution of the membership functions. Non-linear scaling provides a remedy to this problem as it modifies the relative distribution and changes the shape of the membership functions. A common non-linear scaling function for a variable that is symmetric with respect to the origin is of the form:

    $$f(x) = sign(x) \cdot |x|^{\alpha}.$$

    Non-linear scaling increases ($\alpha > 1$) or decreases ($\alpha < 1$) the relative sensitivity in the region around the origin and has the opposite effect at the boundaries of the operating range. Figure 5 depicts some effects caused by the non-linear scaling.

  Derivation methods combining both kinds of context adaptations have been also proposed [23].

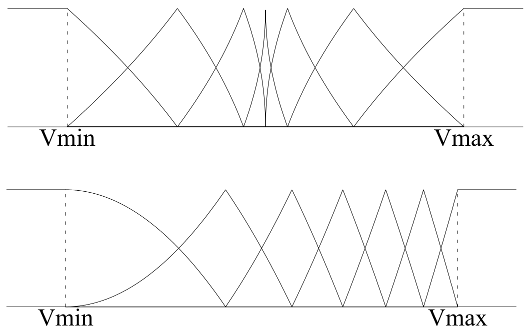

Fig. 5. Non-linear scaling effects

Some other contributions propose methods to define not only the membership function shapes but also the membership function types (such as triangular, trapezoidal, gaussian, and sigmoid) [73].

Interpretability preservation when designing the DB

As said, the DB design may generate intricate semantics that could disturb the expert interpretation, thus losing some legibility degree. Figure 6 illustrates an example where garbled membership functions may involve losing interpretability.

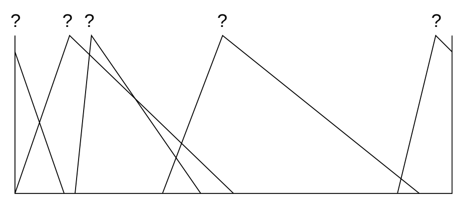

Fig. 6. Certain interpretability may be lost when designing the DB

To decide if a specific DB is interpretable or not is a difficult and subjective task. Nevertheless, some researchers have become aware of this matter and several properties that ensure a good interpretability during the membership function optimization have been proposed [79,80]. Different constrains to the DB design process may be imposed to fulfil all, or some of these properties. A selection of the most important properties and some possible solutions when designing the DB follows:

- *Coverage property* — Every value of the universe of discourse should belong to, at least, a linguistic term. Alternately, a more strict criterion may be considered establishing a minimum level of coverage to be met for the entire universe of discourse.

Possible solutions — This property may be satisfied, e.g., using variation intervals that ensure the overlap between two consecutive fuzzy sets [21] or employing strong fuzzy partitions [17].

- *Normality property* — Every membership function should exhibit full matching with, at least, a value of the universe of discourse. That is, the fuzzy sets should be normal.

  Possible solutions — This property is easily kept forcing the modal points of the extreme membership functions of the variable fuzzy partition to be contained in the universe of discourse.
- *Distinguishability property* — Each linguistic term should have a clear meaning and the associated fuzzy set should clearly define a range of the universe of discourse. In short, the membership functions should be distinct enough from each other.

  Possible solutions — This property may be satisfied, e.g., properly constraining the membership function parameters [54], merging similar membership functions [29], or establishing a semantic order among the linguistic terms [5,6].

Of course, the constrains needed to ensure the semantic integrity make the derivation process less flexible but, moreover of easing the legibility, the risk of overfitting the problem is reduced.

When search techniques are used to design the DB, another possibility to ensure the integrity properties is to include interpretability measures in the objective function, thus guiding the trek to good solutions. Usually, measures of completeness, consistency [45], compactness, or similarity [43] are considered. A more advanced criterion, called conciseness, is proposed in [77] by combining a fuzzy entropy measure, which distinguishes the shapes of the membership functions, with a deviation measure, which evaluates the discrepancy of a membership function from symmetry.

In embedded DB design, another interesting approach is to consider the effect in the RB size caused when defining the number of linguistic terms and their associated membership function shapes. In [68,70], the authors propose to use a simple fuzzy system to select the solution with the desired trade-off between interpretability (number of rules) and accuracy (approximation error) among the different generated KBs.

4.2 Using More Sophisticated RB Learning Methods

These improvements arise as an effort to exploit the accuracy ability of linguistic FRBSs by exclusively focusing on the RB design. In this case, the DB and the model structure keep invariable, thus resulting in the highest interpretability. Usually, all these improvements have the final goal of enhancing the *interpolative reasoning* the FRBS develops. This is one of the most interesting features of FRBSs and plays a key role in their high performance, being a consequence of the cooperative action of the linguistic fuzzy rules. Some specific proposals are explained in the following.

The method COR (cooperative rules) proposed in [12] follows the primary objective of inducing a better cooperation among the linguistic rules. To do that, the RB design is made using global criteria that consider the action of the different rules jointly. It is attained by means of a strong, smart reduction of the search space. The main advantages of the COR methodology are its capability to include heuristic information [14], its flexibility to be used with different metaheuristics [**?**], and its easy integration within other derivation processes [15].

In [32], a method is proposed to refine a previously obtained RB in disjunctive normal form. The heuristic process is comprised of three sequential steps: firstly, the precision of the model is improved by removing some linguistic terms from the rule antecedents; then, the generalization capability is improved by adding linguistic terms to the antecedents; and finally, the completeness of the RB is attained by the addition of new linguistic fuzzy rules.

4.3 Extending the Model Structure

Another way to improve the LFM accuracy is to extend the usual linguistic model structure to make it more flexible. Some specific possibilities are described in the following.

To use linguistic modifiers

A possibility to relax the rule structure is to include certain operators to slightly change the meaning of the linguistic labels involved in the system when necessary [11,20,33]. As Zadeh highlighted in [87], a way to do so without losing the description to a high degree is to use linguistic hedges or, in a wider sense, *linguistic modifiers*. We must remark that the inclusion of linguistic modifiers in fuzzy rules differs from their use to design the DB [52] (explained in Sect. 4.1).

A linguistic modifier [8,9,86] is an operator that alters the membership functions for the fuzzy sets associated to the linguistic labels, giving a more or less precise definition as a result depending on the case. Thus, a new linguistic rule structure arises as follows:

$$\textsf{\textbf{IF}}\ X_1\ \textsf{is}\ \textsf{\textit{lm}}_{X_1}\ A_1\ \textsf{and}\ \ldots\ \textsf{and}\ X_n\ \textsf{is}\ \textsf{\textit{lm}}_{X_n}\ A_n\ \textsf{\textbf{THEN}}\ Y\ \textsf{is}\ \textsf{\textit{lm}}_Y\ B\ ,$$

with lm_{X_i} (lm_Y) being the linguistic modifier to be used (including the identity operator) in the corresponding variable where the membership degree to the linguistic term is given by $\mu_{A_i}^{lm_{X_i}}$ ($\mu_B^{lm_Y}$). An example of a rule with this structure is the following:

$$\textsf{\textbf{IF}}\ X_1\ \textsf{is}\ \textsf{\textit{very}}\ \textsf{high and}\ X_2\ \textsf{is low}\ \textsf{\textbf{THEN}}\ Y\ \textsf{is}\ \textsf{\textit{more-or-less}}\ \textsf{large}\ .$$

Actually, the consideration of linguistic modifiers does not define a new meaning to the so-called *primary terms* – *high*, *low*, and *large* in our example – but they are used as generators whose meaning is defined in the context. In other words, thanks to the *attributed-grammar semantic* [88] associated to the linguistic variables, the final membership functions are computed from the knowledge of the membership functions of the primary terms.

Certainly, the fact of using fuzzy rules with linguistic modifiers will have a significative influence in the behavior of the linguistic FRBS because the matching degree of the rule antecedent as well as the output fuzzy set obtained when applied the implication operator in the inference process are changed.

From a linguistic point of view, the linguistic modifiers are usually classified into those that reinforce the characterizations and those that weaken them. From a FM point of view, however, the most interesting effect caused by the modifiers is the alterations in the inference process (support, center of gravity, etc.) that they cause. From this perspective, we can distinguish among three different kinds of linguistic modifiers:

- *Powered modifiers* [86] — The membership degrees of the linguistic terms are modified with non-linear scaling functions by rising them to the power of some factor (see Fig. 7(a)):

  $$\mu'(x) = (\mu(x))^{\alpha}, \quad 0 < \alpha \ ,$$

  For example, the operator "*very*" squares the membership degree of the linguistic term, i.e., $\mu_T^{very}(x) = (\mu_T(x))^2$.
  With $\alpha < 1$ the modifier dilates the fuzzy set, whilst with $\alpha > 1$ the modifier concentrates the fuzzy set. These kinds of linguistic modifiers are known as *linguistic hedges* [86].
- *Expansive/reducted modifiers* [9,59] — These modifiers change the support and core sets of the fuzzy sets enlarging or reducing them, but trying to keep a center of gravity similar to the original (see Fig. 7(b)). The effect over the fuzzy sets, though similar to the previous one (in terms of dilation and concentration), is rather more severe and it is implemented with linear variations.
- *Shifted modifiers* [9,49] — These kinds of modifiers are defined by translations of the membership functions along their domains (see Fig. 7(c)). The effect over the linguistic terms is more severe than the aforementioned ones.

Powered modifiers (with non linear variation) – linguistic hedges – have an important restriction with respect to expansive/reducted modifiers or shifted modifiers (with linear variation): the support and core sets of the fuzzy sets are not altered. Moreover, when a symmetrical fuzzy set is considered, its center of gravity is not changed. On the contrary, the membership degree of a value to the fuzzy set grows in a non-linear way as it gets closer to the core.

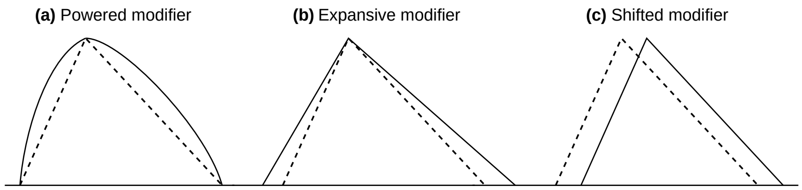

Fig. 7. Types of linguistic modifiers

To use double-consequent rules

This approach involves allowing the RB to present rules where each combination of antecedents may have two consequents associated when it is necessary to improve the model accuracy [22,29,61]. It is clear that this will improve the capability of the model to perform the interpolative reasoning and, thus, its performance. The rule structure obtained will be as follows:

$$\textbf{IF } X_1 \text{ is } A_1 \text{ and } \ldots \text{ and } X_n \text{ is } A_n \textbf{ THEN } Y \text{ is } \{B_1, B_2\} \ .$$

Since each double-consequent fuzzy rule can be decomposed into two different rules with a single consequent, the usual plain fuzzy inference system can be applied. The only restriction imposed is that the defuzzification method must consider the matching degree of the rules fired. For example, the *center of gravity weighted by the matching degree* defuzzification strategy [24] may be used.

When using two consequents per rule, the interpretation of the action performed by every rule may be confusing to some extent. However, we should note this fact does not constitute an inconsistency from the LFM point of view but only a shift of the main labels making the final output of the rule lie in an intermediate zone between both consequents. Indeed, let us suppose that a specific combination of antecedents, "X_1 is A_1 and ... and X_n is A_n," has two different consequents associated, B_1 and B_2. The resulting double-consequent rule may be interpreted as follows [22]:

$$\textbf{IF } X_1 \text{ is } A_1 \text{ and } \ldots \text{ and } X_n \text{ is } A_n \textbf{ THEN } Y \text{ is } \textit{between } B_1 \text{ and } B_2 \ .$$

To use weighted rules

This approach involves using an additional parameter for each rule that indicates its importance degree in the inference process, instead of considering all rules equally important as in the usual case. Thus, more flexibility to improve the interpolative reasoning and, therefore, the model performance, is achieved [1,19,41,62,71,84]. The rule structure will be the following one:

$$\textbf{IF } X_1 \text{ is } A_1 \text{ and } \ldots \text{ and } X_n \text{ is } A_n \textbf{ THEN } Y \text{ is } B \textit{ with } [w] \ ,$$

with w being the real-valued rule weight. In this approach, some changes to the classical inference system must be made to consider the weighted action of each rule.

The operator *with*, which attaches a weight to a rule, may be defined in different ways. One of the most usual options is to multiply the matching degree of the antecedent by the corresponding weight before applying the implication operator, which relates antecedent and consequent. Another possibility is to change the conclusion derived from the implication operator according to the corresponding weight (e.g., changing the support of the obtained fuzzy set).

These weights are usually considered to handle inconsistencies with advanced inference methods [84] or neural networks [19]. Moreover, some proposals make use of them to improve the model accuracy with an automatic learning of weights using different techniques such as heuristic methods [41,71], gradient descent processes [62], or evolutionary algorithms [1].

To use hierarchical knowledge bases

A deeper change in the model structure involves considering hierarchical KBs. In this case, the hierarchical KB is composed of a set of layers where each one contains linguistic partitions with different granularity levels (a layer of the hierarchical DB) and linguistic rules whose linguistic variables take values in these partitions (a layer of the hierarchical RB) [27]. Different learning methods have been proposed to design this extended model structure.

The method proposed in [42] obtains a hierarchical KB by creating several hierarchical linguistic partitions with different granularity levels, generating the complete set of linguistic rules in each of these partitions, taking the union of all of these sets, and finally performing a genetic rule selection process on the whole rule set.

The method introduced in [27] uses an inductive linguistic rule generation method to progressively refine the controversial regions (those covered by linguistic fuzzy rules with a bad performance) by defining new rules in a deeper layer. The obtained hierarchical RB is compacted by a subsequent selection process. Therefore, this latter method follows a *descending* approach refining the regions by increasing the granularity. An *ascending* approach by merging fuzzy sets to progressively reducing the granularity is proposed in [36].

5 Concluding Remarks

The FM research developed in the last two decades was mainly focused on exploiting the flexibility of FM to obtain the maximum accuracy. During this evolution, the derivation methods were improved, the components to be designed were extended, and new model structures were proposed. This search of the accuracy usually set aside the interpretability of the obtained models.

However, we should remember the initial philosophy of fuzzy set theory directed to serve the bridge between the human understanding and the machine processing. In this challenge, the faculty of fuzzy models to express the behavior of the real system in a comprehensible manner acquires a great importance. This is why the current tendency in FM tries to find a better balance between interpretability and accuracy.

This equilibrium is attained from different perspectives. One of the things that attracts the eye is the fact that it is frequently performed by means of previous existing extensions, but used in a more rational and moderate way. Thus, this chapter was aimed to present an introduction to the different tends recently proposed in the specialized literature to improve the accuracy degree of the fuzzy models with the objective of finding the desired trade-off, i.e., preserving their interpretability.

The remaining 14 chapters contained in this volume are excellent works of research in the FM approach studied in this chapter and they properly represent the existing state-of-the-art.

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