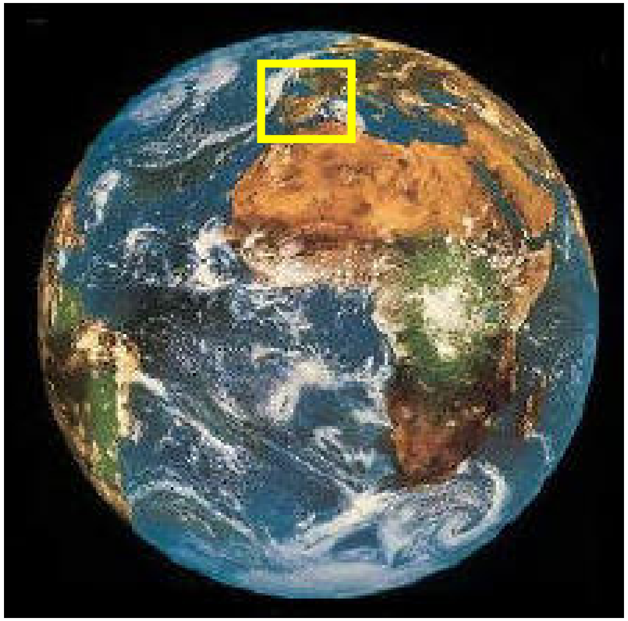
Super resolution and pansharpening of Multispectral Images

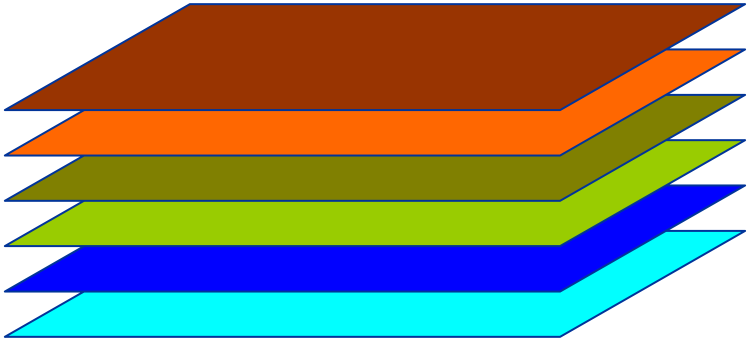
M. Vega, J. Mateos, R. Molina,
Universidad de Granada, Granada, Spain.
A. K. Katsaggelos
Northwestern University, Evanston (IL).

Outline

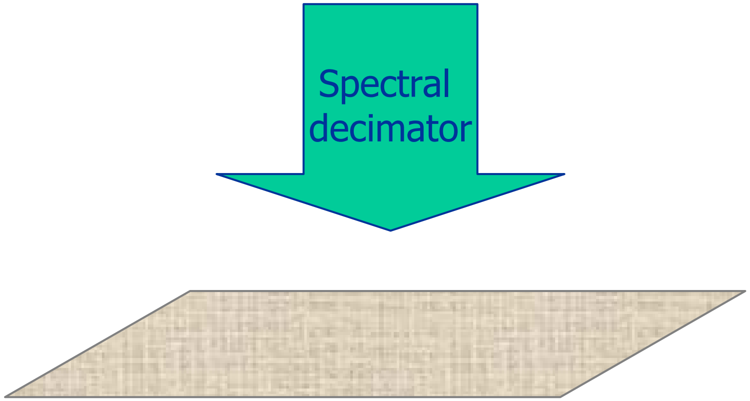
I Super resolution in Remote Sensing

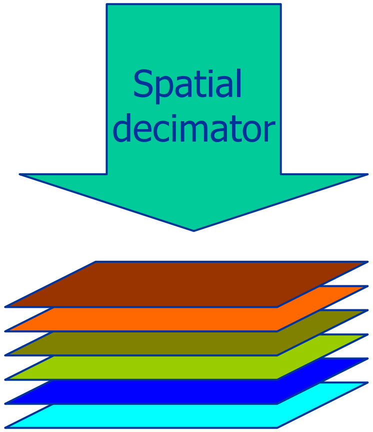


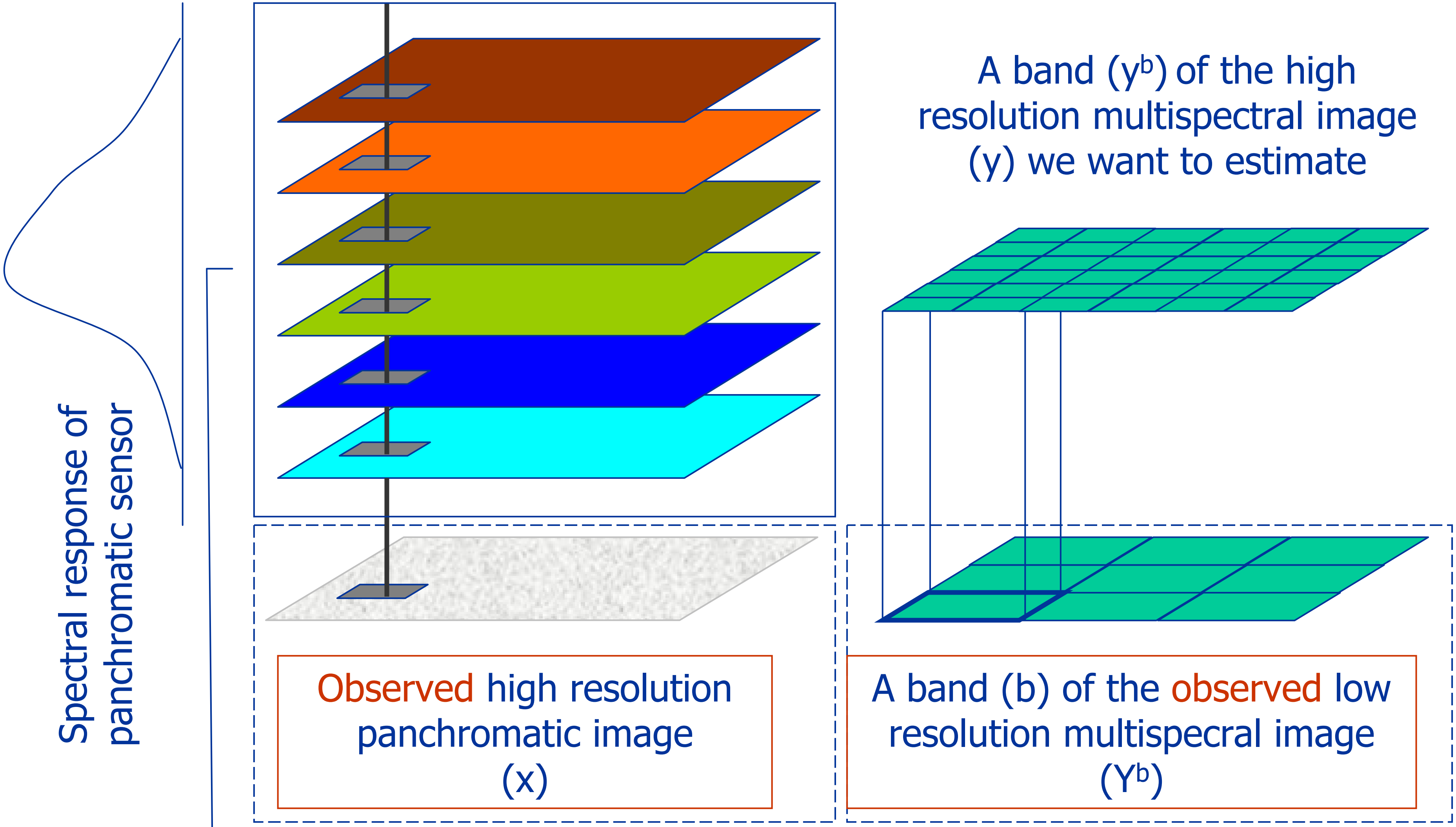
With an ideal sensor we would have high resolution multispectral images.



Unfortunately due to spectral and spatial decimation we have:







Spectral response of panchromatic sensor

A band (y^b) of the high resolution multispectral image (y) we want to estimate

Observed high resolution panchromatic image (x)

A band (b) of the observed low resolution multispecral image (Y^b)

High resolution multispectral image (y) we want to estimate

NOTATION

Upper case: low resolution

Lower case: high resolution

$Y(i)=(Y^1(i),\ldots,Y^K(i))^T$

$y(j)=(y^1(j),\ldots,y^L(j))^T$

II Super resolution methods in Remote Sensing

II.I Method of Akgun et al.

T. Akgun, Y. Altunbasak ana R.M. Mersereau,"Super-resolution Reconstruction of Hyperspectral Images", IEEE Trans. on Image Processing, 2005.

$f^{\lambda}(x)$ denotes a high resolution image at λ wavelength.

$f^{\lambda}(x)$ can be decomposed as

$$f^{\lambda}(x) = \sum_{j=1}^{P} b_j(\lambda) f_j(x)$$

known quantities

High resolution images to be estimated

Comments on the coefficients $b_j(\lambda)$ and the decomposition:

With an example, If P=2 and

$$\lambda \in [0, \lambda_{max}]$$

We could use

$$b_1(\lambda) = \begin{cases} 1 & \text{if } \lambda < \lambda_{\max}/2 \\ 0 & \text{elsewhere} \end{cases} \qquad b_2(\lambda) = \begin{cases} 1 & \text{if } \lambda \geq \lambda_{\max}/2 \\ 0 & \text{elsewhere} \end{cases}$$

and we would have to estimate $f_1(x)$ and $f_2(x)$

We could also use PCA

What observations do we have?

Remember

$$f^{\lambda}(x)=\sum_{j=1}^{P} b_j(\lambda)\, f_j(x)$$

First, each f$^{\lambda}$ is convolved with a convolution filter H producing

Spatial filtering

$$f^{b,\lambda}=Hf^{\lambda}=\sum_{j=1}^{P} b_j(\lambda)\, Hf_j$$

Then, f $^{b,\lambda}$ is weighted (integrated) on λ obtaining

Spectral filtering

$$f^{w,b}=\sum_{\lambda}\eta(\lambda) f^{b,\lambda}=\sum_{\lambda}\eta(\lambda) Hf^{\lambda}=\sum_{\lambda}\eta(\lambda)\left(\sum_{j=1}^{P} b_j(\lambda)\, Hf_j\right)$$

$$=\sum_{j=1}^{P} w_j\; Hf_j$$

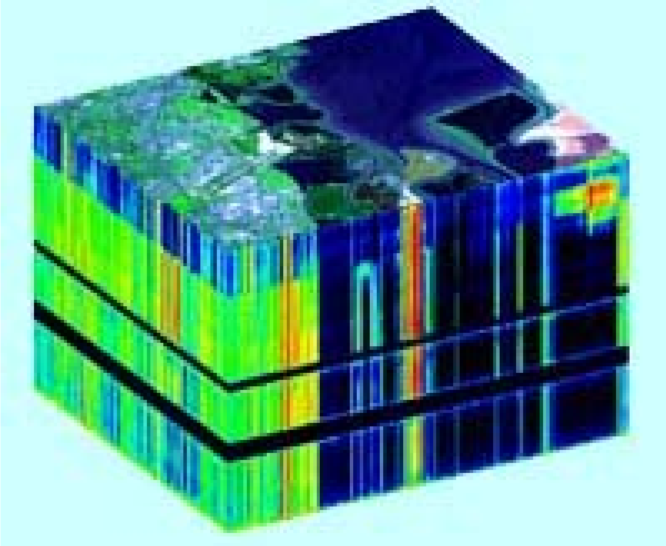
Finally, f ^{w,b} is decimated to produce the observed image

$$g = D(\sum_{\lambda} \eta(\lambda) f^{b,\lambda}) = D\left(\sum_{\lambda} \eta(\lambda) H f^{\lambda} \right)$$

$$= D\left(\sum_{\lambda} \eta(\lambda) \left(\sum_{j=1}^{P} b_j(\lambda) H f_j \right) \right)$$

$$= D\left(\sum_{j=1}^{P} w_j \, H f_j \right) = \sum_{j=1}^{P} w_j \, D H f_j$$

Usually we do not have just one low resolution image but a hypercube of low resolution observations and the above equation can be written for each band g_i of the hypercube. We have



$$g = \begin{pmatrix} g_1 \\ \vdots \\ g_Q \end{pmatrix}$$

where for each image i in the hypercube we have

$$g_i = \sum_{j=1}^{P} w_{i,j} \; DHf_j$$

Gaussian independent noise is usually included in the model so we have

$$g_i = \sum_{j=1}^{P} w_{i,j} \; DHf_j + \varepsilon_i \quad i = 1,\ldots,Q$$

POCS with additional constraints is used to estimate f_j j=1,…,P

Instead of having just one hypercube

$$g = \begin{pmatrix} g_1 \\ \vdots \\ g_Q \end{pmatrix}$$

We may have several hypercubes

$$g^l, \quad l = 1, \ldots, R$$

We could then register all hypercubes with respect to one, for instance g^1, and we would obtain

The weights could be hypercube dependent

hypercube l

The blur could be hypercube dependent

$$g_i^l = \sum_{j=1}^{P} w_{i,j} DHR_{(l,1)} f_j + \varepsilon_i^l \quad i = 1, \ldots, Q$$

Band in hypercube l

Registration between hypercubes l and 1

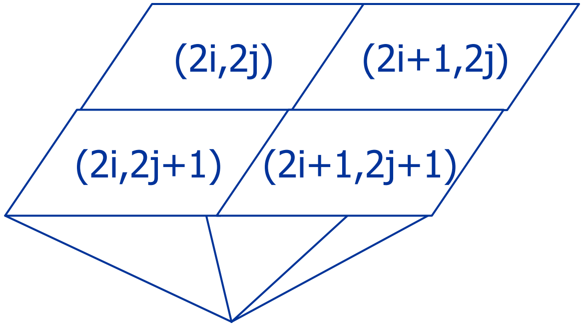
II.II Method of Price

It is assumed that a high resolution panchromatic image x and a low resolution multispectral image Y are available.

We will assume, for simplicity a 2x2 magnifying factor.

(i,j) denotes low resolution pixel. This low resolution pixel consists of four high resolution pixels (u,v) with

$(u,v) \in H_{ij}=\{(2i,2j), (2i+1,2j), (2i,2j+1), (2i+1,2j+1)\}$.



Low resolution pixel (i,j)

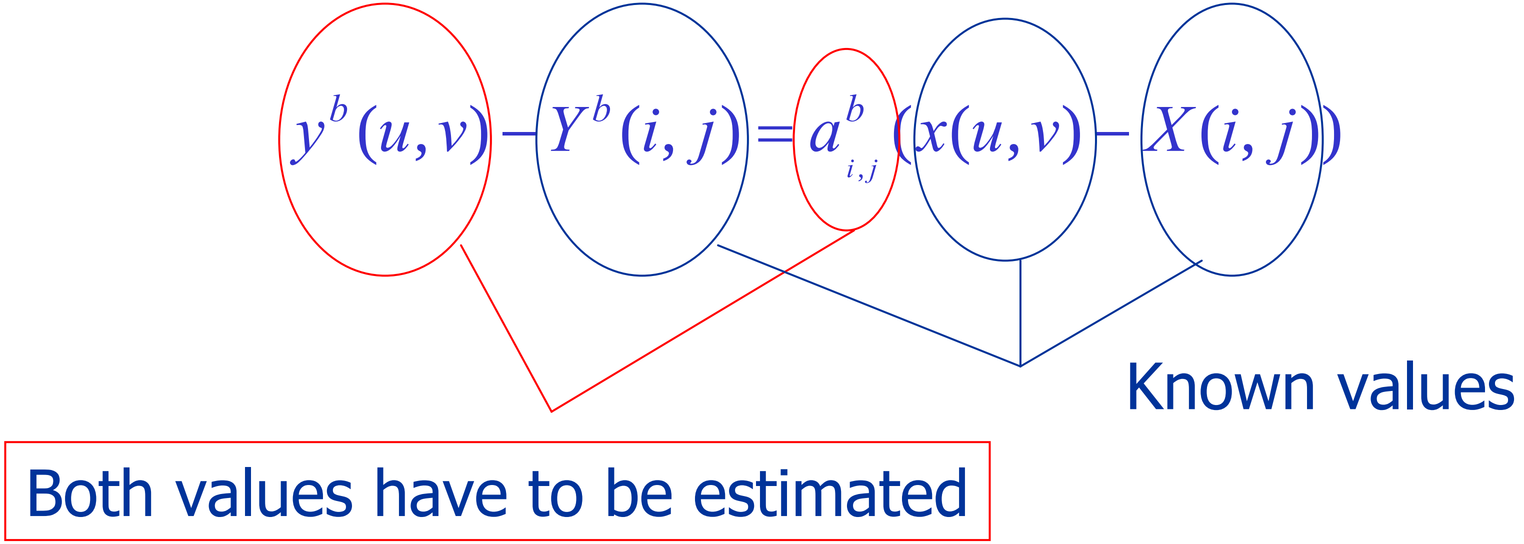
$$X(i,j)=\frac{1}{4}\sum_{(u,v)\in H_{ij}} x(u,v)$$

Panchromatic low resolution image

Price’s model

J.C Price, Combining multispectral Data of Different Spatial Resolution, IEEE Trans. On Geoscience and Remote Sensing, vol. 37, 3, 1199-1203, 1999

For each band b and for (u,v) Є H_{ij} the following assumption is used



$$y^b(u,v) - Y^b(i,j) = a^b_{i,j}\left(x(u,v) - X(i,j)\right)$$

Known values

Both values have to be estimated

First, we estimate $a_{i,j}^b$ and then we calculate $y^b(u,v)$ using the above equation.

Estimating $a_{i,j}^b$ in

$$y^b(u,v) - Y^b(i,j) = a^b_{i,j}(x(u,v) - X(i,j))$$

1. $y^b(u,v)$ and $x(u,v)$ are replaced by $Y^b(p,q)$ and $X(p,q)$, respectively, where $(p,q) \in L_{i,j}$ (a set of low resolution neighboring pixels of pixel (i,j), usually 3x3)
2. and then

$$a^b_{i,j} = \arg\min_a \sum_{(p,q)\in L_{i,j}} (Y^b(p,q) - Y^b(i,j) - a(X(p,q) - X(i,j)))^2$$

which produces

$$a^b_{i,j} =$$

$$\sum_{(u,v)\in L_{i,j}} [Y^b(u,v) - Y^b(i,j)][X(u,v) - X(i,j)] \Big/ \sum_{(u,v)\in L_{i,j}} [X(u,v) - X(i,j)]^2$$

We now estimate $y^b(u,v)$ using

$$y^b(u,v) - Y^b(i,j) = a^b_{i,j}(x(u,v) - X(i,j))$$

Extension:

J.H. Park and M.G. Kang, “Spatially adaptive multi-resolution multispectral image fusion”, INT. J. Remote Sensing, vol. 25, no 23, 5491-5508, 2004.

Let us consider again Price’s equation

$$y^b(u,v) - Y^b(i,j) = a^b_{i,j}\,(x(u,v) - X(i,j))$$

Then, Park and Kang consider for each high resolution pixel (p,q)

$$y^b(p,q) - \bar{y}^b(p,q) = a^b_{u,v}(x(p,q) - \bar{x}(p,q))$$

where

$$\bar{\mathrm{x}}(\mathrm{p},\mathrm{q}) = \frac{1}{4}\sum_{(\mathrm{u},\mathrm{v})\ \text{neighbors of}\ (\mathrm{p},\mathrm{q})} \mathrm{x}(\mathrm{u},\mathrm{v}) \qquad \bar{\mathrm{y}}^{\mathrm{b}}(\mathrm{p},\mathrm{q}) = \frac{1}{4}\sum_{(\mathrm{u},\mathrm{v})\ \text{neighbors of}\ (\mathrm{p},\mathrm{q})} \mathrm{y}^{\mathrm{b}}(\mathrm{u},\mathrm{v})$$

In order to estimate $a_{u,v}^{b}$ in

$$y^{b}(p,q)-\bar{y}^{b}(p,q)=a_{u,v}^{b}(x(p,q)-\bar{x}(p,q))$$

The following procedure is proposed:

- $x(p,q)$ is replaced by its downsampled (to the size of the low resolution images) and then upsampled version to its original size. The new value is denoted by $x'(u,v)$. Then $\bar{x}'(u,v)$ is calculated.
- $y^{b}(u,v)$ is estimated as an upsampled version of Y^{b}. The new value is denoted y'^{b} and $\bar{y}'^{b}$ is now calculated.

and $a_{u,v}^{b}$ is estimated from

$$y'^{b}(p,q)-\bar{y}'^{b}(p,q)=a_{u,v}^{b}(x'(p,q)-\bar{x}'(p,q))$$

as

$$a^{b}_{p,q} = \arg\min_{a} \sum_{(u,v)\text{ neighbors of (p,q)}} w_{u,v}(y'^{b}(u,v) - \bar{y}'^{b}(u,v) - a(x'(u,v) - \bar{x}'(u,v)))^{2}$$

where $w_{u,v}$ is a similarity measure between pixels (u,v) and (p,q).

Finally, $y^b(p,q)$ is calculated using

$$y^{b}(p,q) = a^{b}_{p,q}(x(p,q) - \bar{x}(p,q)) + \bar{y}'^{b}(p,q)$$

II.III Method of Eismann et at

The panchromatic high resolution image x can be written as

$$x=S^t y+\eta$$

where y is the high resolution multispectral image we want to estimate, S is a sparse matrix whose rows are the spectral response functions for the panchromatic pixel locations and η is the noise. The above equation produces $P(x|y)$.

The low resolution observations Y can be expressed as

$$Y=Hy+\varepsilon$$

where ε is the noise and H is a sparse matrix whose rows are the spatial response functions for the low resolution hyperspectral pixels. The above equation produces $P(Y|y)$.

Using the Bayesian paradigm, our goal becomes finding the *Maximum a Posteriori (MAP),* that is

$$\hat{y} = \arg\ \max_y\ P(y \mid x, Y)$$

where we have

$$P(y \mid x, Y) \propto P(y) P(x, Y \mid y)$$

assuming independence between x and Y given y we write

$$P(y \mid x, Y) \propto \boxed{P(y) P(x \mid y)} P(Y \mid y)$$

or

$$P(y \mid x, Y) \propto \boxed{P(y \mid x)} P(Y \mid y)$$

The only remaining task is the definition of P(y) or the conditional distribution P(y|x) depending on the model we want to use.

Using the model

$$P(y \mid x, Y) \propto P(y \mid x) P(Y \mid y)$$

R.C. Hardie, M.T. Eismannand G.L. Wilson, ‘MAP estimation for Hyperspectral Image Resolution Enhancement Using an Auxiliary Sensor’, IEEE Trans. on Image Processing, vol. 13, no 9, pp 1174-1184, 2004.

The authors propose to estimate P(y|x) using a joint Gaussian distribution for (y,x) and then calculate the conditional.

Mean and Covariance matrices are obtained from the panchromatic and low resolution images. Covariance matrices are improved by the use of clustering techniques.

Using the model

$$P(y \mid x, Y) \propto P(y)P(x \mid y)P(Y \mid y)$$

M.T. Eismann and and R.C. Hardie, 'Application of the Stochastic Mixing Model to Hyperspectral Resolution Enhancement', IEEE Trans. on Geoscience and Remote Sensing, vol. 42, no 9, pp 1924-1933, 2004.

P(y) is estimated for each pixel as a mixture of Gaussian distributions and the mean and covariance of each member of the mixture is estimated using the Stochastic Mixing Model (SMM), see paper for details. The element of the mixture with the highest probability defines then the prior model.

Note that we can also use the SMM when estimating P(y|x).

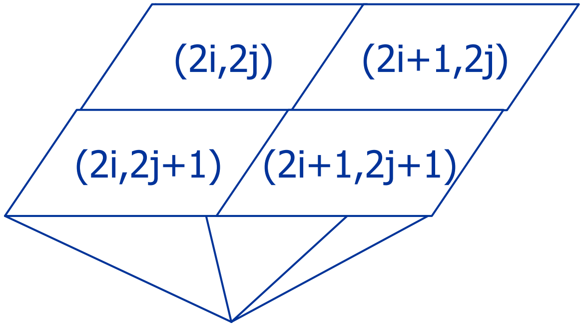
M.T. Eismann and and R.C. Hardie, 'Hyperspectral Resolution Enhancement Using High_resolution Multispectral Imaginary with arbitray response functions', IEEE Trans. on Geoscience and Remote Sensing, vol. 43, no 3, pp 455-465, 2005.

III. A new SR model in Remote Sensing

We assume that a high resolution panchromatic image x and a low resolution multispectral image Y are available. We want to obtain a high resolution hypercube y.

(i, j) denotes low resolution pixel. This low resolution pixel consists of four high resolution pixels (u,v) with

(u,v) Є H_{ij}={(2i,2j), (2i+1,2j), (2i,2j+1), (2i+1,2j+1)}.



Low resolution pixel (i,j)

$$Y^b(i,j) = \frac{1}{4} \sum_{(u,v)\in H_{ij}} y^b(u,v) = (Hy^b)(i,j)$$

Low resolution band from its corresponding high resolution band

Let us assume that we have B observed low resolution images $Y^1,\ldots,Y^B$ and a high resolution panchromatic image x.

We want to estimate the corresponding B high resolution images $y^1,\ldots,y^B$ with the use of the information provided by the low resolution observations and the panchromatic image.

Let us denote by y the whole set of high resolution images $y^1,\ldots,y^B$ we want to estimate.

The process to obtain the low resolution observations for the high resolution images we want to estimate is modeled by

$$P(Y^b \mid y) = P(Y^b \mid y^b) \propto \exp\left[-\frac{\alpha_b}{2} \| Y^b - DHy^b \|^2 \right]$$

where D models the downsampling operation

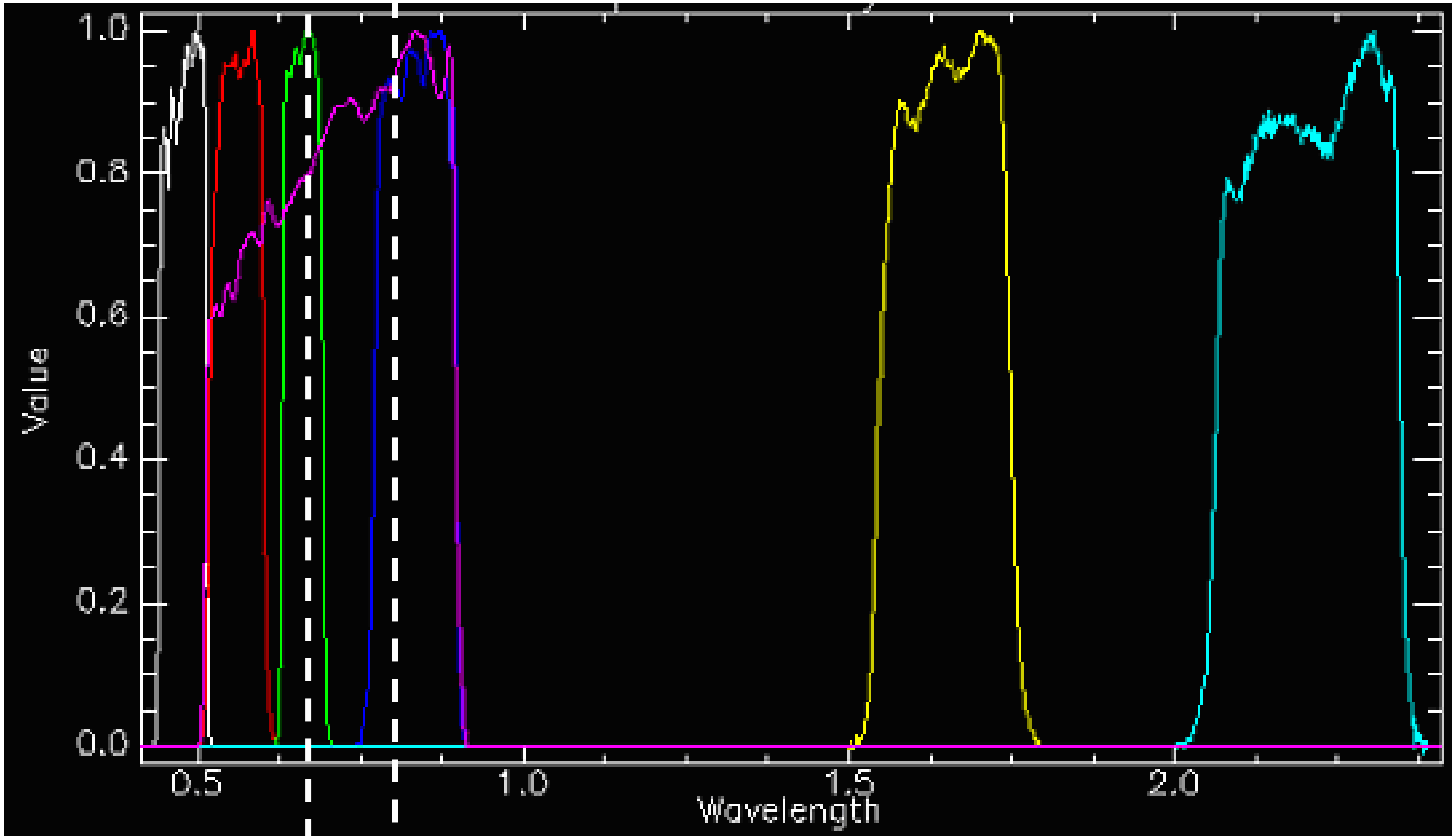
The panchromatic image is formed as a linear combination of the high resolution hypercube bands plus additive noise:

$$x(u,v)=\sum_{b}\lambda^{b}y^{b}(u,v)+\varepsilon(u,v)$$

$\lambda^b \geq 0$ are *known* quantities weighting the contribution of each high resolution band we want to estimate to the high resolution panchromatic image.

There is work to be done on the estimation of these weights. Blind deconvolution techniques?

Lansat ETM+ Spectral Response



Color	LANDSAT ETM+ band
White	1 (0.45 µm to 0.515 µm)
Red	2 (0.525 µm to 0.605 µm)
Green	3 (0.63 µm to 0.69 µm)
Blue	4 (0.75 µm to 0.9 µm)

Color	LANDSAT ETM+ band
Yellow	5 (1.55 µm to 1.75 µm)
Not shown	6 (10.4 µm to 12.5 µm)
Cyan	7 (2.08 µm to 2.35 µm)
Magenta	Pan (0.51 µm to 0.9 µm)

LANDSAT ETM+ band	λ^b
1 (0.45 µm to 0.515 µm)	0.015606
2 (0.525 µm to 0.605 µm)	0.22924
3 (0.63 µm to 0.69 µm)	0.25606
4 (0.75 µm to 0.9 µm)	0.49823
5 (1.55 µm to 1.75 µm)	0.0
7 (2.08 µm to 2.35 µm)	0.0

The panchromatic image provides no information on these two bands

We intend to reconstruct all the B bands y^b , $b=1,\dots,B$ simultaneously. For Lansat ETM+ images we have three bands to be reconstructed. So, in this case B=4.

We assume

$$P(x \mid y^1,\dots,y^B) \propto \exp\left[-\frac{\alpha}{2} \| x - \sum_{j=1}^{B} \lambda^j y^j \|^2\right]$$

A priori we assume that all high resolution images are smooth and no correlation between them exists (this needs more work), so we write

$$P(y) = \prod_{b=1}^{B} P\left(y^{b}\right) \propto \prod_{b=1}^{B} \exp\left[-\frac{\beta_b}{2} \| C y^{b} \|^{2}\right]$$

where C denotes the Laplacian operator.

We now use the Bayesian paradigm and write

$$P(y \mid x, Y) \propto P(y)P\left(x, Y \mid y\right)$$

Not very realistic

$$= \left(\prod_{b=1}^{B} P\left(y^{b}\right)\right) P\left(x, Y \mid y\right)$$

$$= \left(\prod_{b=1}^{B} P\left(y^{b}\right)\right) P(x \mid y) P\left(Y \mid y\right)$$

$$= \left(\prod_{b=1}^{B} P\left(y^{b}\right)\right) P(x \mid y) \left(\prod_{b=1}^{B} P\left(Y^{b} \mid y^{b}\right)\right)$$

Our goal then becomes finding

$$\hat{y} = \arg\max_{y} \left(\prod_{b=1}^{B} P\left(y^{b}\right)\right) P(x \mid y) \left(\prod_{b=1}^{B} P\left(Y^{b} \mid y^{b}\right)\right)$$

$$\hat{y} = \arg\min_{y} \left\{ \sum_{b} \underbrace{\alpha_b \left\| Y^b - DHy^b \right\|^2}_{\text{Fidelity to low resolution observations}} + \underbrace{\alpha \left\| x - \sum_{j} \lambda^j y^j \right\|^2}_{\text{Fidelity to the panchromatic image}} + \sum_{b} \underbrace{\beta_b \left\| Cy^b \right\|^2}_{\text{Smoothness constraints}} \right\}$$

Because of the form of the function to be optimized (of the involved matrices), its solution can be found using non-iterastive techniques.

Note also that the unknown parameters can be estimated using the E-M algorithm (work in progress).

IV. Examples

panchromatic

Low resolution bands 1 to 4

band 1

Low resolution bilinearly interpolated

Price´s method

Proposed method

band 2

Low resolution bilinearly interpolated

Price´s method

Proposed method

band 3

Low resolution bilinearly interpolated

Price´s method

Proposed method

band 4

Low resolution bilinearly interpolated

Price´s method

Proposed method

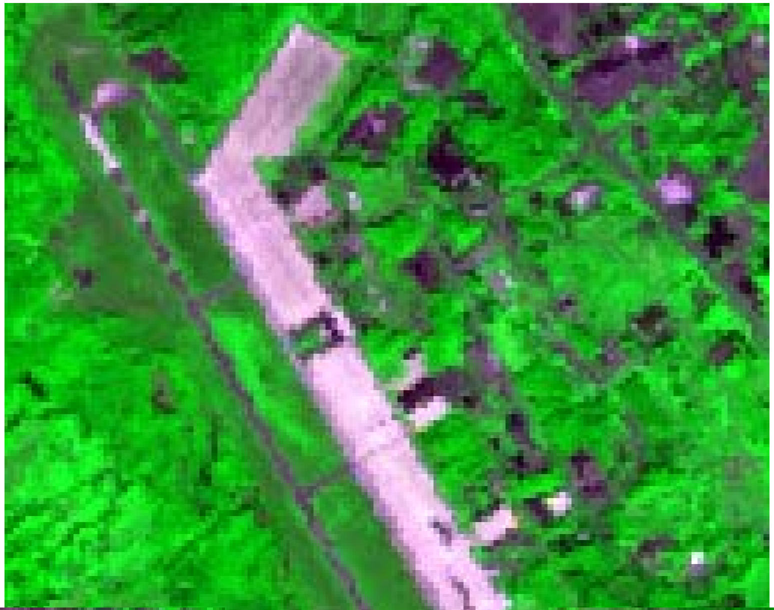
R = band 3
G = band 2
B = band 1

Low resolution bilinearly interpolated

Price´s method

Proposed method

R = band 3
G = band 4
B = band 2



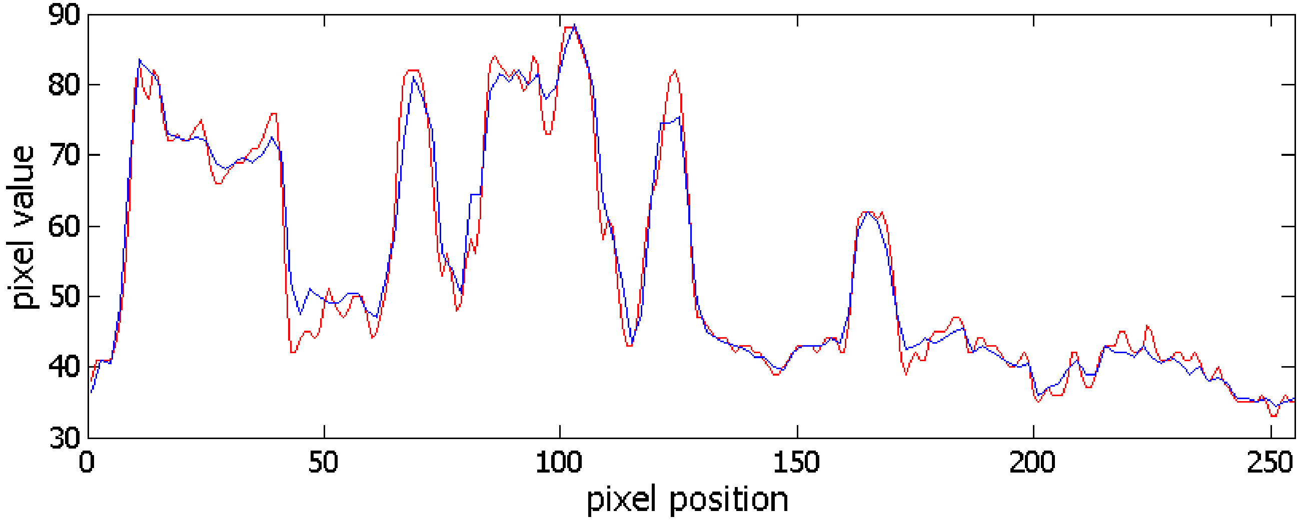
Low resolution bilinearly interpolated

Price´s method

Proposed method

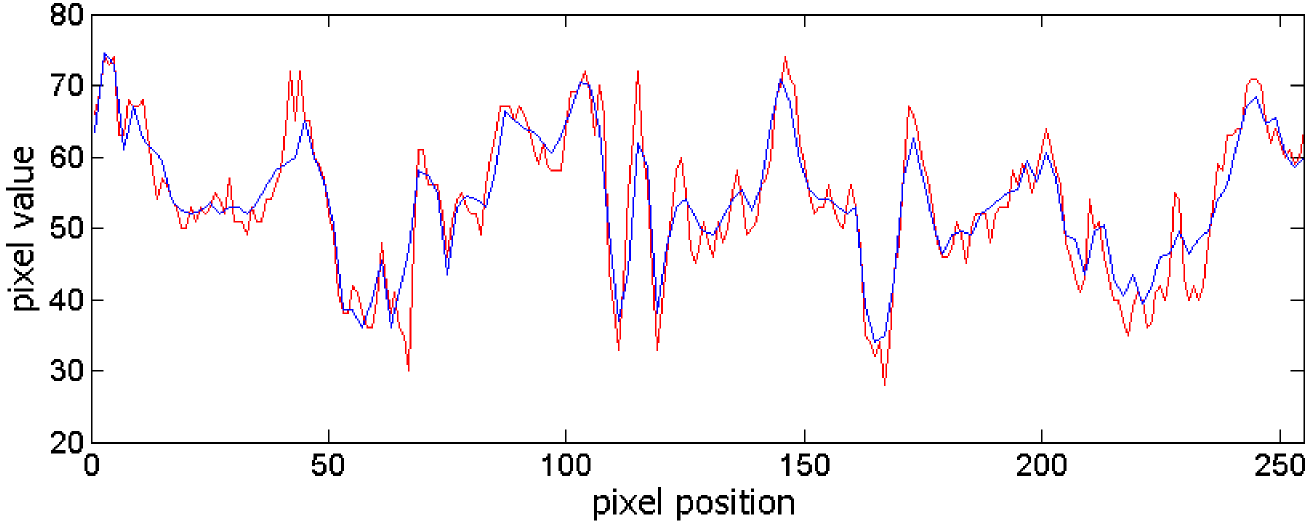
— Low resolution band 1

— Reconstructed band 1



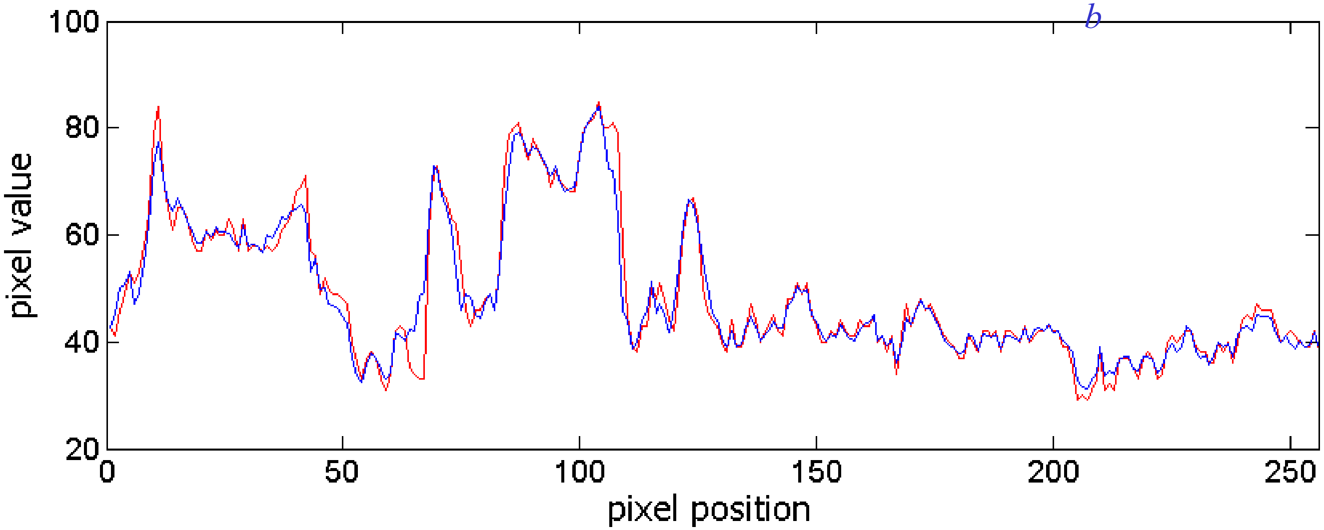
Low resolution band 4

Reconstructed band 4

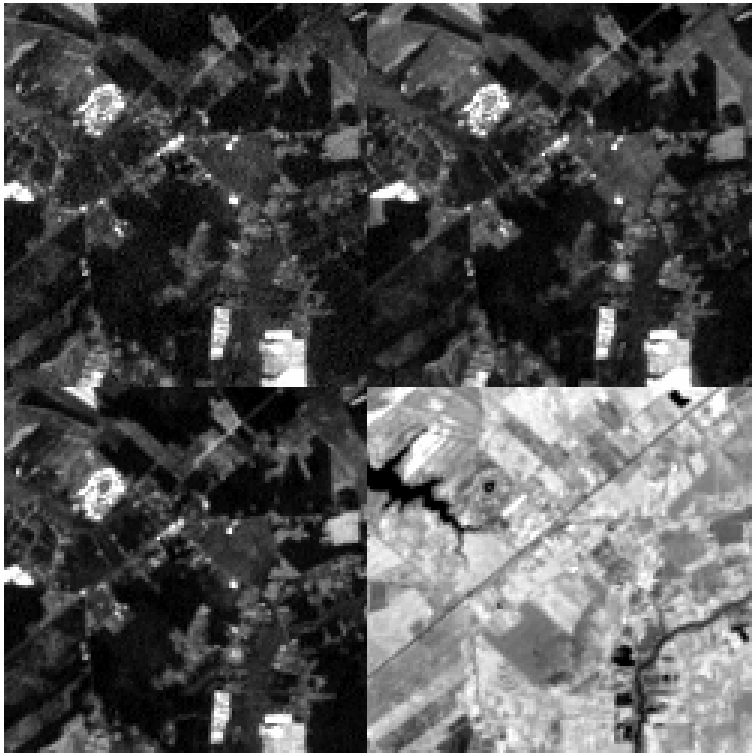


Panchromatic

$\sum_{b} \lambda^{b} \hat{y}^{b}(u,v)$

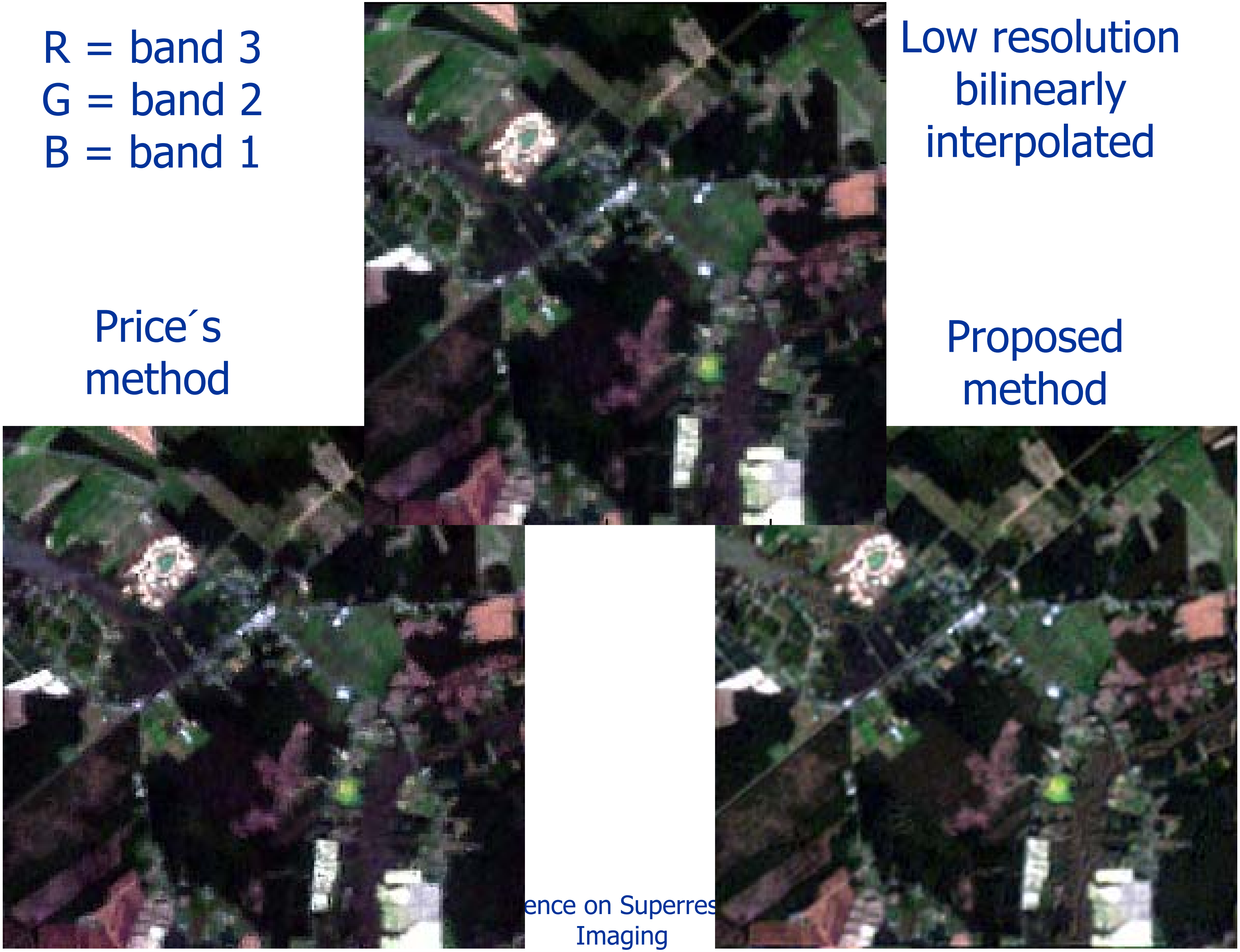


panchromatic



Low resolution bands 1 to 4

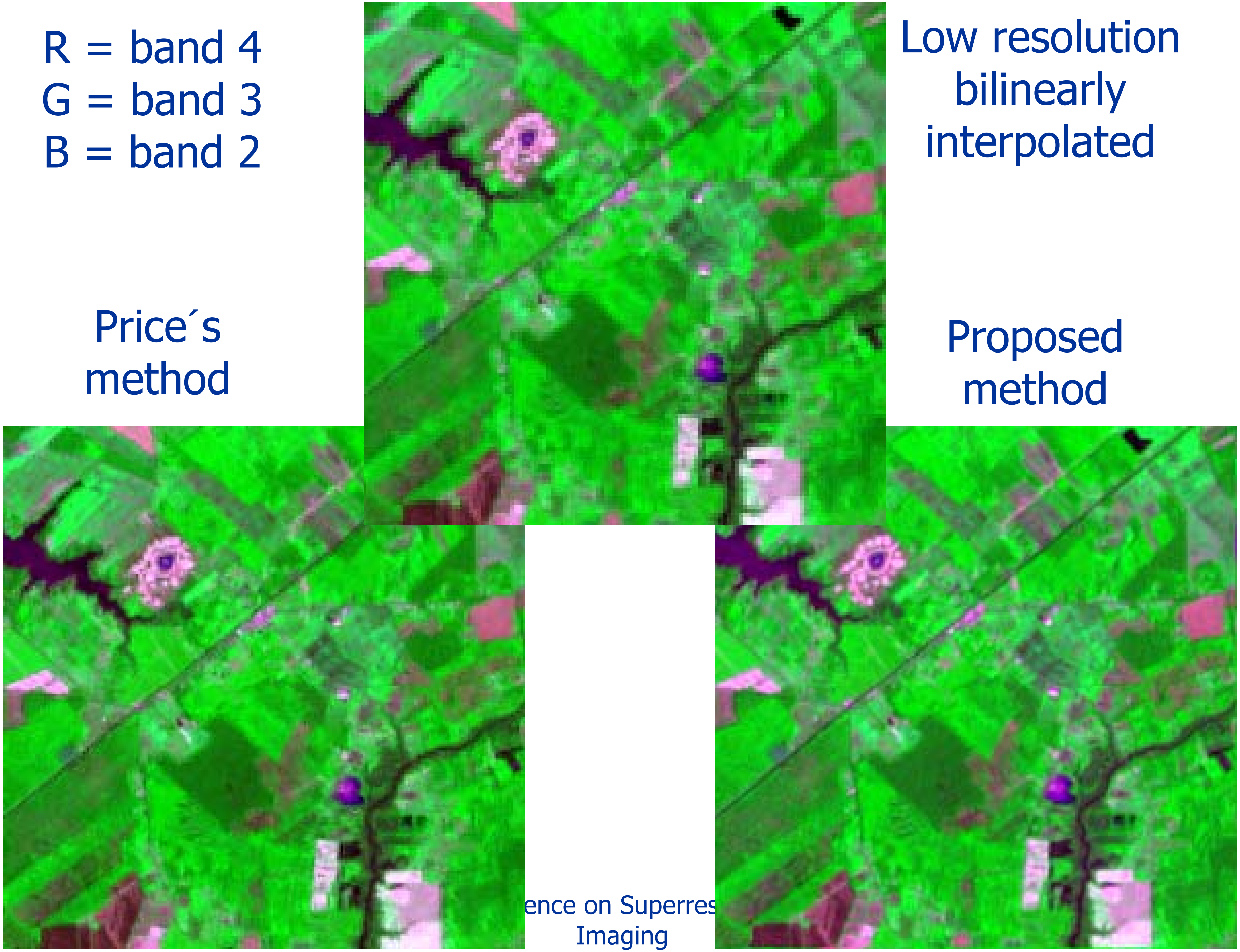
R = band 3
G = band 2
B = band 1



Low resolution bilinearly interpolated

Price´s method

Proposed method



R = band 4
G = band 3
B = band 2

Low resolution bilinearly interpolated

Price´s method

Proposed method

V. Conclusions

Super resolution methods in Remote Sensing have been described.

A new super resolution method in Remote Sensing has been proposed.

Some preliminary examples have been shown.