BAYESIAN PARTIAL OUT-OF-FOCUS BLUR REMOVAL WITH PARAMETER ESTIMATION

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ABSTRACT

In this paper we propose a novel partial out-of-focus blur removal method developed within the Bayesian framework. We concentrate on the removal of background out-of-focus blurs that are present in the images in which there is a strong interest to keep the foreground in sharp focus. However, often there is a desire to recover background details out of such partially blurred image. In this work, a non-convex l_p-norm prior with $0 < p < 1$ is used as the background and foreground image prior and a total variation (TV) based prior is utilized for both the background blur and the occlusion mask, that is, the mask determining the pixels belonging to the foreground. In order to model transparent foregrounds, the values in the occlusion mask are assumed to belong to the closed interval $[0,1]$. The proposed method is derived by utilizing bounds on the priors for the background and foreground image, the background blur and the occlusion mask using the majorization-minimization principle. Maximum *a posteriori* Bayesian inference is performed and as a result, the background and foreground image, the background blur, the occlusion mask and the model parameters are simultaneously estimated. Experimental results are presented to demonstrate the advantage of the proposed method over the existing ones.

1. INTRODUCTION

It is not uncommon for the different objects or regions within the original scene to move independently throughout the scene by taking their own directions and velocities. Therefore, often the blurring function spatially varies throughout the image. Another example of a spatially varying blur is the out-of-focus blur that is widely used to accentuate the depth of field during image acquisition. In general, this type of blurring does not fit well into the single-layer image formation model (e.g., [1]), and consequently more advanced image formation models are needed. The most common way to remove the spatially varying blur is to consider acquiring multiple images of the original scene [2, 3]. In some recent methods a redesign of the imaging system is proposed. For example, in [4] a fluttered shutter camera is proposed to achieve a broader frequency response compared to the traditional imaging system. Note that the method in [4] still requires knowledge of the moving object boundaries and object velocities to successfully restore a partially blurred image. In addition, a parabolic camera is proposed in [5, 6] in order to minimize the information loss for 1D and 2D constant-velocity motion, respectively. The restoration of partially blurred images acquired by the traditional imaging systems is still of great importance given that the majority of imaging devices are traditional ones. For the reminder of the section we concentrate on a two-layer image formation model. The restoration method proposed in this paper is based on a single-image observation, and therefore multiple captures of the original scene are not required.

The standard formulation of the degradation model for the partially blurred images in which only one layer is degraded is given in matrix-vector form by (please see [7] for more details)

$$\mathbf{y} = \mathbf{H}_f(\mathbf{o} \odot \mathbf{x}_f) + (\mathbf{H}_b\mathbf{x}_b) \odot (\mathbf{1} - \mathbf{H_f o}) + \mathbf{n}, \qquad (1)$$

where the $N \times 1$ vectors $\mathbf{x}_f$, $\mathbf{x}_b$, $\mathbf{y}$, $\mathbf{o}$ and $\mathbf{n}$ represent respectively the foreground image, the background image, the available noisy and partially blurred image, the occlusion mask and the noise with independent elements of variance $\sigma_{\mathbf{n}}^2 = \beta^{-1}$, while matrices $\mathbf{H}_f$ and $\mathbf{H}_b$ represent the blurring matrices created from the blurring point spread functions ($\mathbf{h}_f$ and $\mathbf{h}_b$) of the foreground and background, respectively. The images are assumed to be of size $m \times n = N$, and they are lexicographically ordered into $N \times 1$ vectors. The operator $\odot$ denotes the Hadamard product and $\mathbf{1}$ denotes the $N \times 1$ column vector with all components equal to one. Given $\mathbf{y}$, the partially blurred image restoration problem calls for finding estimates of $\mathbf{x}_f$, $\mathbf{x}_b$, $\mathbf{H}_f$, $\mathbf{H}_b$ and $\mathbf{o}$ using prior knowledge on them.

This paper is organized as follows. In Section 2 we provide the proposed Bayesian modeling of the partially blurred image restoration problem. The Bayesian inference is presented in Section 3. Experimental results are provided in Section 4 and conclusions drawn in Section 5.

2. BAYESIAN MODELING

The observation noise is modeled as a zero mean white Gaussian random vector. Therefore, the observation model is defined as

$$\mathrm{p}(\mathbf{y}|\beta, \mathbf{x}_f, \mathbf{h}_f, \mathbf{x}_b, \mathbf{h}_b, \mathbf{o}) \propto \beta^{N/2} \exp\left[-\frac{\beta}{2}\|\mathbf{y} - \mathbf{H}_f(\mathbf{o} \odot \mathbf{x}_f) - (\mathbf{H}_b\mathbf{x}_b) \odot (\mathbf{1} - \mathbf{H_f o})\|^2\right], \qquad (2)$$

where β is the precision of the multivariate Gaussian distribution.

In this work we assume that the foreground blur is available to us. We make this assumption since partially blurred out-of-focus images are often encountered in portrait photography (i.e., the foreground blur is a unit impulse) in which the foreground is of very high visual quality. In general this assumption works well in practice as can be seen in Section 4.

In summary, the foreground blur $\mathbf{h}_f$ is assumed to be known (i.e., unit impulse), while the background image, the foreground image, the background blur, the occlusion mask and the model parameters are assumed to be unknown. Consequently we can simplify the observation model defined in (2) with

$$\mathrm{p}(\mathbf{y}|\beta,\mathbf{x}_f,\mathbf{h}_f,\mathbf{x}_b,\mathbf{h}_b,\mathbf{o}) \propto \beta^{N/2}\exp\left[-\frac{\beta}{2}\|\mathbf{y}-\mathbf{D}_o\mathbf{x}_f-(\mathbf{I}-\mathbf{D}_o)\mathbf{H}_b\mathbf{x}_b\|^2\right], \quad (3)$$

where $\mathbf{I}$ denotes the identity matrix and $\mathbf{D}_o$ denotes a diagonal matrix defined as $\mathbf{D}_o = \mathrm{diag}(\mathbf{o})$.

For the background image prior we utilize a variant of the generalized Gaussian distribution, given by

$$\mathrm{p}(\mathbf{x}_b|\alpha) = \frac{1}{Z_{\mathrm{GG}}(\alpha)}\exp\left[-\sum_{d\in\mathrm{D}}\alpha_d\sum_i|\Delta_i^d(\mathbf{x}_b)|^p\right], \quad (4)$$

where $Z_{\mathrm{GG}}(\alpha)$ is the partition function, $0<p<1$, α denotes the set $\{\alpha_d\}$ and $d\in\mathrm{D}=\{h,v,hh,vv,hv\}$. $\Delta_i^h(\mathbf{u})$ and $\Delta_i^v(\mathbf{u})$ correspond to, respectively, the horizontal and vertical first order differences, at pixel i, that is, $\Delta_i^h(\mathbf{u}) = u_i - u_{l(i)}$ and $\Delta_i^v(\mathbf{u}) = u_i - u_{a(i)}$, where $l(i)$ and $a(i)$ denote the nearest neighbors of i, to the left and above, respectively. The operators $\Delta_i^{hh}(\mathbf{u})$, $\Delta_i^{vv}(\mathbf{u})$, $\Delta_i^{hv}(\mathbf{u})$ correspond to, respectively, horizontal, vertical and horizontal-vertical second order differences, at pixel i.

In order to eliminate the need to estimate each α_d we assume that $\alpha_h=\alpha_v=\alpha$ and $\alpha_{hh}=\alpha_{vv}=\alpha_{hv}=\alpha/2$. Additionally, similarly to [8], the partition function will be approximated as $Z_{\mathrm{GG}}(\alpha)\propto\alpha^{-\lambda_1 N/p}$, where λ_1 is a positive real number. We then simplify (4) accordingly to obtain the following background image prior

$$\mathrm{p}(\mathbf{x}_b|\alpha_b) \propto \alpha_b^{\lambda_1 N/p}\exp\left[-\alpha_b\sum_{d\in\mathrm{D}}2^{1-o(d)}\sum_i|\Delta_i^d(\mathbf{x}_b)|^p\right], \quad (5)$$

where $o(d)\in\{1,2\}$ denotes the order of the difference operator $\Delta_i^d(\mathbf{x}_b)$. Similarly, for the foreground image we assume the following prior

$$\mathrm{p}(\mathbf{x}_f|\alpha_f) \propto \alpha_f^{\lambda_2 N/p}\exp\left[-\alpha_f\sum_{d\in\mathrm{D}}2^{1-o(d)}\sum_i|\Delta_i^d(\mathbf{x}_f)|^p\right], \quad (6)$$

where λ_2 is a positive real number. For the background-image blur we utilize a total-variation prior given by

$$\mathrm{p}(\mathbf{h}_b|\gamma_b) \propto \gamma_b^{\lambda_3 N}\exp\left[-\gamma_b\mathrm{TV}(\mathbf{h}_b)\right], \quad (7)$$

where λ_3 is a positive real number and $\mathrm{TV}(\mathbf{h}_b)$ is defined as

$$\mathrm{TV}(\mathbf{h}_b) = \sum_i\sqrt{(\Delta_i^h(\mathbf{h}_b))^2+(\Delta_i^v(\mathbf{h}_b))^2}. \quad (8)$$

Similarly, for the occlusion mask we utilize a total-variation prior,

$$\mathrm{p}(\mathbf{o}|\gamma_o) \propto \gamma_o^{\lambda_4 N}\exp\left[-\gamma_o\mathrm{TV}(\mathbf{o})\right], \quad (9)$$

where λ_4 is a positive real number and $\mathrm{TV}(\mathbf{o})$ is defined as

$$\mathrm{TV}(\mathbf{o}) = \sum_i\sqrt{(\Delta_i^h(\mathbf{o}))^2+(\Delta_i^v(\mathbf{o}))^2}. \quad (10)$$

In this work we use flat improper hyperpriors for each unknown hyperparameter, that is, we utilize

$$\mathrm{p}(\omega)\propto\mathrm{const}, \quad (11)$$

where $\omega\in\{\beta,\alpha_b,\alpha_f,\gamma_b,\gamma_o\}$.

3. BAYESIAN INFERENCE

Bayesian inference on the unknown components of the partially blurred image restoration problem is based on the estimation of the unknown posterior distribution $\mathrm{p}(\alpha_b,\alpha_f,\beta,\gamma_b,\gamma_o,\mathbf{x}_b,\mathbf{h}_b,\mathbf{o},\mathbf{x}_f\mid\mathbf{y},\mathbf{h}_f)$, given by

$$\mathrm{p}(\alpha_b,\alpha_f,\beta,\gamma_b,\gamma_o,\mathbf{x}_b,\mathbf{h}_b,\mathbf{o},\mathbf{x}_f\mid\mathbf{y},\mathbf{h}_f) = \frac{\mathrm{p}(\alpha_b,\alpha_f,\beta,\gamma_b,\gamma_o,\mathbf{x}_b,\mathbf{h}_b,\mathbf{o},\mathbf{y},\mathbf{x}_f,\mathbf{h}_f)}{\mathrm{p}(\mathbf{y},\mathbf{h}_f)}. \quad (12)$$

In this work, we adopt the maximum *a posteriori* (MAP) approach to obtain a single point $(\bar{\alpha_b},\bar{\alpha_f},\bar{\beta},\bar{\gamma_b},\bar{\gamma_o},\bar{\mathbf{x}_b},\bar{\mathbf{x}_f},\bar{\mathbf{h}_b},\bar{\mathbf{o}})$ estimate, denoted as $\bar{\Theta}$, that maximizes $\mathrm{p}(\alpha_b,\alpha_f,\beta,\gamma_b,\gamma_o,\mathbf{x}_b,\mathbf{x}_f,\mathbf{h}_b,\mathbf{o}\mid\mathbf{y},\mathbf{h}_f)$ as follows,

$$\begin{aligned}\bar{\Theta} &= \underset{\Theta}{\mathrm{argmax}}\,\mathrm{p}(\alpha_b,\alpha_f,\beta,\gamma_b,\gamma_o,\mathbf{x}_b,\mathbf{x}_f,\mathbf{h}_b,\mathbf{o}\mid\mathbf{y},\mathbf{h}_f)\\ &= \min_{\Theta}\Bigg\{\frac{\beta}{2}\|\mathbf{y}-\mathbf{D}_o\mathbf{x}_f-(\mathbf{I}-\mathbf{D}_o)\mathbf{H}_b\mathbf{x}_b\|^2+\\ &\quad+\alpha_b\sum_{d\in\mathrm{D}}2^{1-o(d)}\sum_i|\Delta_i^d(\mathbf{x}_b)|^p+\alpha_f\sum_{d\in\mathrm{D}}2^{1-o(d)}\sum_i|\Delta_i^d(\mathbf{x}_f)|^p+\\ &\quad+\gamma_b\mathrm{TV}(\mathbf{h}_b)+\gamma_o\mathrm{TV}(\mathbf{o})-\frac{\lambda_1 N}{p}\log\alpha_b-\frac{\lambda_2 N}{p}\log\alpha_f\\ &\quad-\frac{N}{2}\log\beta-\lambda_3 N\log\gamma_b-\lambda_4 N\log\gamma_o\Bigg\}.\end{aligned} \quad (13)$$

As can be seen from (13), obtaining the point estimate that maximizes the posterior distribution $\mathrm{p}(\alpha_b,\alpha_f,\beta,\gamma_b,\gamma_o,\mathbf{x}_b,\mathbf{x}_f,\mathbf{h}_b,\mathbf{o}\mid\mathbf{y},\mathbf{h}_f)$ is not straightforward since it requires the minimization of a non-convex functional. Note that maximizing the posterior distribution $\mathrm{p}(\alpha_b,\alpha_f,\beta,\gamma_b,\gamma_o,\mathbf{x}_b,\mathbf{x}_f,\mathbf{h}_b,\mathbf{o}\mid\mathbf{y},\mathbf{h}_f)$ with the maximum *a posteriori* approach is equivalent to the variational Bayesian based maximization (see [9]) for the example case when all the posterior distributions are assumed to be degenerate.

In this paper, we resort to a majorization-minimization approach to bound the non-convex image prior $\mathrm{p}(\mathbf{x}_b|\alpha_b)$ by the functional $\mathrm{M}_1(\alpha_b,\mathbf{x}_b,\mathbf{V})$, that is

$$\mathrm{p}(\mathbf{x}_b|\alpha_b)\geq\mathrm{const}\cdot\mathrm{M}_1(\alpha_b,\mathbf{x}_b,\mathbf{V}). \quad (14)$$

The majorization-minimization approach has been utilized in several approaches for image restoration [9, 10].

The functional $\mathrm{M}_1(\alpha_b,\mathbf{x}_b,\mathbf{V})$ is derived by considering the relationship between the weighted geometric and arithmetic means, which is given by

$$z^{p/2}v^{1-p/2}\leq\frac{p}{2}z+\left(1-\frac{p}{2}\right)v, \quad (15)$$

where z, v and p are positive real numbers. We first rewrite (15) as

$$z^{p/2} \leq \frac{p}{2} \frac{z + \frac{2-p}{p} v}{v^{1-p/2}}. \quad (16)$$

Using (16) we obtain

$$|\Delta_i^d(\mathbf{x}_b)|^p \leq \frac{p}{2} \frac{[\Delta_i^d(\mathbf{x}_b)]^2 + \frac{2-p}{p} v_{d,i}}{v_{d,i}^{1-p/2}}. \quad (17)$$

Therefore we have

$$\mathrm{p}(\mathbf{x}_b|\alpha_b) = \mathrm{const} \cdot \alpha_b^{\lambda_1 N/p} \exp\left[-\alpha_b \sum_{d \in \mathrm{D}} 2^{1-o(d)} \sum_i |\Delta_i^d(\mathbf{x}_b)|^p\right]$$

$$\geq \mathrm{const} \cdot \alpha_b^{\lambda_1 N/p} \exp\left[-\frac{\alpha_b p}{2} \sum_{d \in \mathrm{D}} 2^{1-o(d)} \sum_i \frac{[\Delta_i^d(\mathbf{x}_b)]^2 + \frac{2-p}{p} v_{d,i}}{v_{d,i}^{1-p/2}}\right] \quad (18)$$

and so $\mathrm{M}_1(\alpha_b, \mathbf{x}_b, \mathbf{V})$ is defined as

$$\mathrm{M}_1(\alpha_b, \mathbf{x}_b, \mathbf{V}) = \alpha_b^{\lambda_1 N/p} \exp\left[-\frac{\alpha_b p}{2} \sum_{d \in \mathrm{D}} 2^{1-o(d)} \sum_i \frac{[\Delta_i^d(\mathbf{x}_b)]^2 + \frac{2-p}{p} v_{d,i}}{v_{d,i}^{1-p/2}}\right], \quad (19)$$

where $\mathbf{V}$ is a matrix with elements $v_{d,i}$. Following the same approach, the foreground image prior can be bounded by

$$\mathrm{p}(\mathbf{x}_f|\alpha_f) \geq \mathrm{const} \cdot \mathrm{M}_2(\alpha_f, \mathbf{x}_f, \mathrm{B}). \quad (20)$$

where B is a matrix with elements $b_{d,i} > 0$ and $\mathrm{M}_2(\alpha_f, \mathbf{x}_f, \mathrm{B})$ is defined as

$$\mathrm{M}_2(\alpha_f, \mathbf{x}_f, \mathrm{B}) = \alpha_f^{\lambda_2 N/p} \exp\left[-\frac{\alpha_f p}{2} \sum_{d \in \mathrm{D}} 2^{1-o(d)} \sum_i \frac{[\Delta_i^d(\mathbf{x}_f)]^2 + \frac{2-p}{p} b_{d,i}}{b_{d,i}^{1-p/2}}\right]. \quad (21)$$

Similarly, the majorization-minimization criterion is used to bound the blur prior $\mathrm{p}(\mathbf{h}_b|\gamma_b)$ utilizing the functional $\mathrm{M}_3(\gamma_b, \mathbf{h}_b, \mathbf{u})$. Let us define, for γ_b and any $N-$dimensional vector $\mathbf{u} \in (R^+)^N$, with components $u_i,\ i = 1, \ldots, N$, the following functional

$$\mathrm{M}_3(\gamma_b, \mathbf{h}_b, \mathbf{u}) = \gamma_b^{\lambda_3 N} \exp\left[-\frac{\gamma_b}{2} \sum_i \frac{(\Delta_i^h(\mathbf{h}_b))^2 + (\Delta_i^v(\mathbf{h}_b))^2 + u_i}{\sqrt{u_i}}\right]. \quad (22)$$

Using the inequality in (16) with $p = 1$, for $z \geq 0$ and $v > 0$

$$\sqrt{z} \leq \sqrt{v} + \frac{1}{2\sqrt{v}}(z - v), \quad (23)$$

we obtain

$$\mathrm{p}(\mathbf{h}_b|\gamma_b) \geq \mathrm{const} \cdot \mathrm{M}_3(\gamma_b, \mathbf{h}_b, \mathbf{u}). \quad (24)$$

Once again, the majorization-minimization criterion is used to bound the occlusion mask total variation prior $\mathrm{p}(\mathbf{o}|\gamma_o)$ utilizing the functional $\mathrm{M}_4(\gamma_o, \mathbf{o}, \mathbf{z})$, such that,

$$\mathrm{p}(\mathbf{o}|\gamma_o) \geq \mathrm{const} \cdot \mathrm{M}_4(\gamma_o, \mathbf{o}, \mathbf{z}), \quad (25)$$

where

$$\mathrm{M}_4(\gamma_o, \mathbf{o}, \mathbf{z}) = \gamma_o^{\lambda_4 N} \exp\left[-\frac{\gamma_o}{2} \sum_i \frac{(\Delta_i^h(\mathbf{o}))^2 + (\Delta_i^v(\mathbf{o}))^2 + z_i}{\sqrt{z_i}}\right], \quad (26)$$

and $\mathbf{z} \in (R^+)^N$ is an $N-$dimensional vector.

The lower bounds of $\mathrm{p}(\mathbf{x}_b|\alpha_b)$, $\mathrm{p}(\mathbf{x}_f|\alpha_f)$, $\mathrm{p}(\mathbf{h}_b|\gamma_b)$ and $\mathrm{p}(\mathbf{o}|\gamma_o)$ defined above lead to the following lower bound of the distribution $\mathrm{p}(\alpha_b, \alpha_f, \beta, \gamma_b, \gamma_o, \mathbf{x}_b, \mathbf{h}_b, \mathbf{o}, \mathbf{y}, \mathbf{x}_f, \mathbf{h}_f)$,

$$\begin{aligned}&\mathrm{p}(\alpha_b, \alpha_f, \beta, \gamma_b, \gamma_o, \mathbf{x}_b, \mathbf{h}_b, \mathbf{o}, \mathbf{y}, \mathbf{x}_f, \mathbf{h}_f) = \mathrm{p}(\alpha_b)\mathrm{p}(\alpha_f)\mathrm{p}(\beta)\mathrm{p}(\gamma_b)\\ &\mathrm{p}(\gamma_o)\mathrm{p}(\mathbf{x}_b|\alpha_b)\mathrm{p}(\mathbf{x}_f|\alpha_f)\mathrm{p}(\mathbf{h}_b|\gamma_b)\mathrm{p}(\mathbf{o}|\gamma_o)\mathrm{p}(\mathbf{y}|\beta, \mathbf{x}_b, \mathbf{h}_b, \mathbf{x}_f, \mathbf{h}_f, \mathbf{o}) \geq\\ &\mathrm{const} \cdot \mathrm{M}_1(\alpha_b, \mathbf{x}_b, \mathbf{V})\mathrm{M}_2(\alpha_f, \mathbf{x}_f, \mathrm{B})\mathrm{M}_3(\gamma_b, \mathbf{h}_b, \mathbf{u})\mathrm{M}_4(\gamma_o, \mathbf{o}, \mathbf{z})\\ &\qquad \mathrm{p}(\mathbf{y}|\beta, \mathbf{x}_b, \mathbf{h}_b, \mathbf{x}_f, \mathbf{h}_f, \mathbf{o}).\end{aligned}$$

Therefore, a single point estimate that maximizes a lower bound of the posterior distribution $\mathrm{p}(\alpha_b, \alpha_f, \beta, \gamma_b, \gamma_o, \mathbf{x}_b, \mathbf{x}_f, \mathbf{h}_b, \mathbf{o} \mid \mathbf{y}, \mathbf{h}_f)$ is found as follows

$$\begin{aligned}\bar{\Theta} = \min_{\Theta} \Bigg\{ &\frac{\beta}{2} \|\mathbf{y} - \mathbf{D}_o \mathbf{x}_f - (\mathbf{I} - \mathbf{D}_o)\mathbf{H}_b \mathbf{x}_b\|^2 +\\ &\frac{\alpha_b p}{2} \sum_{d \in \mathrm{D}} 2^{1-o(d)} \sum_i \frac{[\Delta_i^d(\mathbf{x}_b)]^2 + \frac{2-p}{p} v_{d,i}}{v_{d,i}^{1-p/2}} +\\ &\frac{\alpha_f p}{2} \sum_{d \in \mathrm{D}} 2^{1-o(d)} \sum_i \frac{[\Delta_i^d(\mathbf{x}_f)]^2 + \frac{2-p}{p} b_{d,i}}{b_{d,i}^{1-p/2}} +\\ &\frac{\gamma_b}{2} \sum_i \frac{(\Delta_i^h(\mathbf{h}_b))^2 + (\Delta_i^v(\mathbf{h}_b))^2 + u_i}{\sqrt{u_i}} +\\ &\frac{\gamma_o}{2} \sum_i \frac{(\Delta_i^h(\mathbf{o}))^2 + (\Delta_i^v(\mathbf{o}))^2 + z_i}{\sqrt{z_i}} +\\ &-\frac{\lambda_1 N}{p} \log \alpha_b - \frac{\lambda_2 N}{p} \log \alpha_f +\\ &-\frac{N}{2} \log \beta - \lambda_3 N \log \gamma_b - \lambda_4 N \log \gamma_o \Bigg\}.\end{aligned} \quad (27)$$

Using (27) for all unknowns in an alternating fashion, we obtain the final algorithm as shown below.

Algorithm. *Given $\alpha_b^1, \alpha_f^1, \beta^1, \gamma_b^1, \gamma_o^1, \mathbf{o}^1, \mathbf{u}^1, \mathbf{z}^1, \mathbf{x}_f^1, \mathrm{B}^1$ and $\mathbf{V}^1$, where the rows of $\mathbf{V}^k$ and B^k are denoted by $\mathbf{v}_d^k \in (R^+)^N$ and $\mathbf{b}_d^k \in (R^+)^N$ respectively, with $d \in \{h, v, hh, vv, hv\}$ and initial estimate of the blurring filter $\mathbf{h}_b^1$.*

For $k = 1, 2, \ldots$ until a stopping criterion is met:

1. *Calculate*

$$\mathbf{x}_b^k = \left[\beta^k (\mathbf{D}_b^k \mathbf{H}_b^k)^t (\mathbf{D}_b^k \mathbf{H}_b^k) + \alpha_b^k p \sum_d 2^{1-o(d)} (\Delta^d)^t \mathbf{W}_d^k (\Delta^d)\right]^{-1} \times \beta^k (\mathbf{D}_b^k \mathbf{H}_b^k)^t (\mathbf{y} - \mathbf{D}_o^k \mathbf{x}_f^k), \quad (28)$$

where $\mathbf{W}_d^k$ is a diagonal matrix with entries $\mathbf{W}_d^k(i,i) = (v_{d,i}^k)^{p/2-1}$, $\mathbf{D}_o^k$ is a diagonal matrix defined as $\mathbf{D}_o^k = diag(\mathbf{o}^k)$ and $\mathbf{D}_b^k = \mathbf{I} - \mathbf{D}_o^k$.

2. *Calculate*

$$\mathbf{h}_b^{k+1} = \left[\beta^k (\mathbf{D}_b^k \mathbf{X}_b^k)^t (\mathbf{D}_b^k \mathbf{X}_b^k) + \gamma_b^k \sum_{d \in \{h,v\}} (\Delta^d)^t \mathbf{U}^k (\Delta^d)\right]^{-1} \times \beta^k (\mathbf{D}_b^k \mathbf{X}_b^k)^t (\mathbf{y} - \mathbf{D}_o^k \mathbf{x}_f^k), \quad (29)$$

where $\mathbf{U}^k$ *is a diagonal matrix with entries* $\mathbf{U}^k(i,i) = (u_i^k)^{-1/2}$ *and* $\mathbf{X}_b^k$ *denotes a convolution matrix created from the background image estimate* $\mathbf{x}_b^k$*.*

3. *Calculate*

$$\mathbf{o}^{k+1} = \left[\beta^k (\mathbf{H}_o^{k+1})^t (\mathbf{H}_o^{k+1}) + \gamma_o^k \sum_{d \in \{h,v\}} (\Delta^d)^t \mathbf{Z}^k (\Delta^d)\right]^{-1} \times \beta^k (\mathbf{H}_o^{k+1})^t (\mathbf{y} - \mathbf{H}_b^{k+1} \mathbf{x}_b^k), \quad (30)$$

where $\mathbf{Z}^k$ *and* $\mathbf{H}_o^{k+1}$ *are diagonal matrices defined as* $\mathbf{Z}^k(i,i) = diag((z_i^k)^{-1/2})$ *and* $\mathbf{H}_o^{k+1} = diag(\mathbf{x}_f^k) - diag(\mathbf{H}_b^{k+1}\mathbf{x}_b^k)$*.*

4. *Calculate*

$$\mathbf{x}_f^{k+1} = \Big[\beta^k (\mathbf{D}_o^{k+1})^t (\mathbf{D}_o^{k+1}) \quad (31)$$

$$+\alpha_f^k p \sum_d 2^{1-o(d)} (\Delta^d)^t \mathbf{W}_d^k (\Delta^d)\Big]^{-1} \times \beta^k (\mathbf{D}_o^{k+1})^t (\mathbf{y} - \mathbf{D}_b^{k+1} \mathbf{H}_b^{k+1} \mathbf{x}_b^k), \quad (32)$$

where $\mathbf{W}_d^k$ *is a diagonal matrix with entries* $\mathbf{W}_d^k(i,i) = (b_{d,i}^k)^{p/2-1}$*.*

5. *For each* $d \in \{h, v, hh, vv, hv\}$ *calculate*

$$v_{d,i}^{k+1} = [\Delta_i^d(\mathbf{x}_b^k)]^2, \quad (33)$$

$$b_{d,i}^{k+1} = [\Delta_i^d(\mathbf{x}_f^k)]^2, \quad (34)$$

6. *Calculate*

$$u_i^{k+1} = [\Delta_i^h(\mathbf{h}_b^{k+1})]^2 + [\Delta_i^v(\mathbf{h}_b^{k+1})]^2, \quad (35)$$

$$z_i^{k+1} = [\Delta_i^h(\mathbf{o}^{k+1})]^2 + [\Delta_i^v(\mathbf{o}^{k+1})]^2, \quad (36)$$

7. *Calculate*

$$\alpha_b^{k+1} = \frac{\lambda_1 N/p}{\sum_{d \in \mathrm{D}} 2^{1-o(d)} \sum_i |\Delta_i^d(\mathbf{x}_b^k)|^p}, \quad (37)$$

$$\alpha_f^{k+1} = \frac{\lambda_2 N/p}{\sum_{d \in \mathrm{D}} 2^{1-o(d)} \sum_i |\Delta_i^d(\mathbf{x}_f^{k+1})|^p}, \quad (38)$$

$$\beta^{k+1} = \frac{N}{\| \mathbf{y} - \mathbf{D}_o^{k+1}\mathbf{x}_f^{k+1} - \mathbf{D}_b^{k+1}\mathbf{H}_b^{k+1}\mathbf{x}_b^k \|^2}, \quad (39)$$

$$\gamma_b^{k+1} = \frac{\lambda_3 N}{\mathrm{TV}(\mathbf{h}_b^{k+1})}, \quad (40)$$

$$\gamma_o^{k+1} = \frac{\lambda_4 N}{\mathrm{TV}(\mathbf{o}^{k+1})}. \quad (41)$$

Set restored image, $\hat{\mathbf{x}}$*,*

$$\hat{\mathbf{x}} = \lim_{k \to \infty} \{\mathbf{D}_o^{k+1}\mathbf{x}_f^{k+1} + (\mathbf{I} - \mathbf{D}_o^{k+1})\mathbf{x}_b^k\}. \quad (42)$$

In this work we set the values of the parameters p, λ_1, λ_2, λ_3, and λ_4 equal to 0.8, 0.5 1.0 0.5 and 0.5, respectively. The robustness of the proposed method will be tested and evaluated by restoring the partially blurred images photographed by a commercial camera. Additionally, since the proposed algorithm is initialized with the unit impulse as the initial background blur estimate, it is particularly important in the first few iterations to keep parameters α_b and γ_b relatively high compared to the parameter β. This procedure prevents the proposed algorithm from converging to the undesirable background blur estimate of unit impulse.

4. EXPERIMENTAL RESULTS

In this section we present experimental results obtained with the use of the proposed algorithm. For all experiments, the initial background blur was set to the unit impulse, and the proposed algorithm is terminated if the termination criterion $\|\mathbf{x}_b^k - \mathbf{x}_b^{k-1}\| / \|\mathbf{x}_b^{k-1}\| < 10^{-4}$ is satisfied or if the maximum number of iterations reaches 100. After updating the unknowns, as described in Section 3, we apply known domain constraints and truncate the background blur estimate, the background and foreground image (both images are normalized) estimate and the occlusion mask to an interval $[0,1]$. In addition, the occlusion mask is initialized with a user defined trimap (available at alphamatting.com) and the blur estimate is normalized (after truncation) so that the sum of its elements equals one. A trimap is a popular way [11, 12] to initialize the alpha matting algorithms in which the black color defines a clear background, the white color defines a clear foreground and the gray color defines an unknown region of the occlusion mask that has to be estimated. Some examples of the user defined trimap are shown in Figure 1.

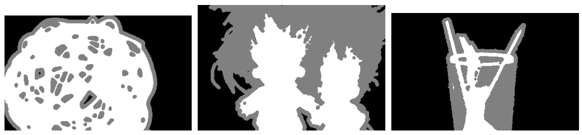

Figure 1: Example of initial estimates of the occlusion mask (trimaps from alphamatting.com).

We evaluate the performance of the proposed method on the images taken by a commercial camera. The test images used in this section are available online (www.alphamatting.com). In [13], the authors compared different partial blur removal methods [14–16]. In this work we consider for the comparison the best performing method reported in [13]. Example restorations obtained by the proposed algorithm are shown in Figure 2. It is clear from Figure 2 that the proposed algorithm provides restorations with high visual quality that are very competitive with the existing state-of-the art method proposed in [13].

5. CONCLUSIONS

In this paper a novel partially blurred image restoration algorithm is presented. More specifically, the focus of this paper was to remove partial out-of-focus background blur that is often present in images with sharp in focus foreground. The proposed algorithm was developed within a Bayesian framework utilizing an l_p-norm based sparse prior for the background and foreground image, and a total-variation prior for both the background blur and the occlusion mask. Restora-

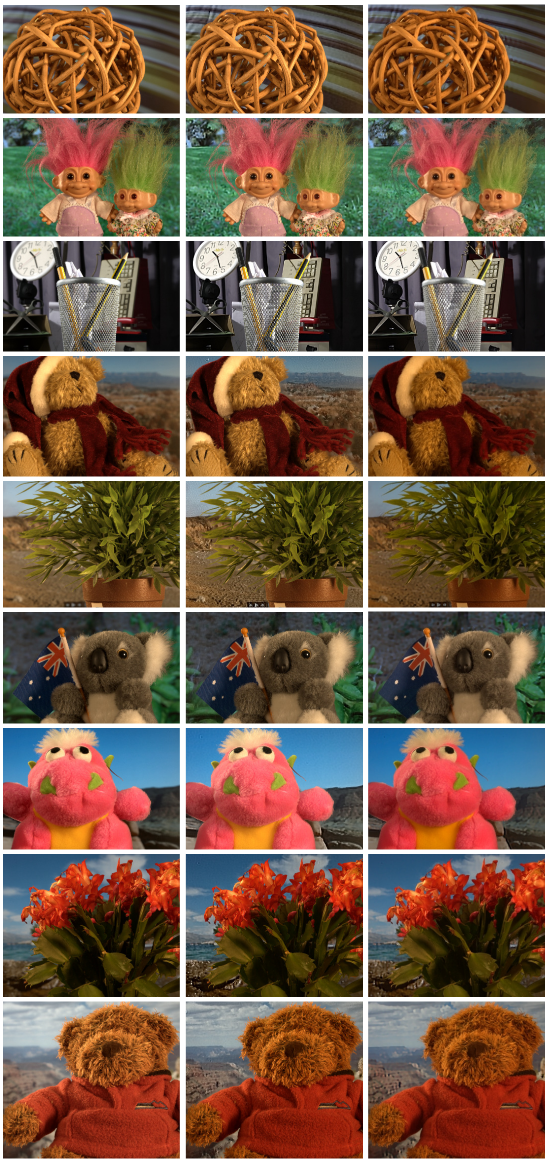

Figure 2: Example restorations from alphamatting.com: 1st column represents nine different partially blurred observations, 2nd column represents restorations obtained by the proposed algorithm, 3rd column represents restorations obtained by the method proposed in [13].

tions of the partially blurred images taken by a commercial camera demonstrate that using sparse priors and the proposed parameter estimation can substantially improve the quality of the observed partially blur image. Finally, it was shown that the performance of the proposed algorithm is higher than existing state-of-the-art blind partially blurred image restoration algorithms. Future work includes extending the proposed method for the restoration of partially blurred images in which the foreground part of the observed image is not in sharp focus (e.g., when the foreground is in motion).

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