THE INSTITUTE OF ELECTRICAL AND ELECTRONICS ENGINEERS INC.

SIGNAL PROCESSING
ALGORITHMS, ARCHITECTURES, ARRANGEMENTS, AND APPLICATIONS
SPA 2013
September 26-28th, 2013, Poznań, POLAND

General Shearlet Pansharpening Method using Bayesian Inference

Israa Amro
Computer Information Systems Dept.
Al-Quds Open University
Hebron, Palestine
Email: iamro@correo.ugr.es

Javier Mateos
Departamento de Ciencias de la Computación e I.A.
Universidad de Granada
Granada, Spain
Email: jmd@decsai.ugr.es

***Abstract*—Pansharpening is a technique that fuses the information of a low resolution multispectral image and a high resolution panchromatic image, usually remote sensing images, to produce a high resolution multispectral image. In the literature, this task has been addressed from different points of view being one of the most popular the wavelets and contourlet based algorithms. Recently, the shearlet transform, has been proposed. This transform combines the advantages of the wavelets and contourlet transform, with a more efficient directional information representation. In this paper we propose a new shearlet based pansharpening algorithm that generalizes a number of pansharpening approaches and compare it with contourlet based and shearlet based methods. The performance of the proposed shearlet based method is assessed numerically and visually for synthetic and SPOT images.**

***Index Terms*—Pansharpening, Remote sensing, shearlet transform, Bayesian Inference.**

I. INTRODUCTION

Earth observing satellites usually have optical sensors that provide a panchromatic (PAN) image with high spatial and low spectral resolution, and a multispectral (MS) image with high spectral and low spatial resolution. Due to physical and technological constrains, they can not provide high spectral and high spatial resolution in a single image. Pansharpening is a pixel based technique that fuses a low resolution multispectral (LRMS) image and a high resolution PAN image to provide a high resolution multispectral (HRMS) image with the spatial resolution and quality of PAN image and the spectral resolution of the MS image. In the literature, this task has been addressed from different points of view (see [1] for a complete review).

In recent years, multiresolution-based methods, which includes Laplacian pyramid [2], wavelet-based methods [3] and contourlet-based methods [4]–[6], are becoming popular. The basic idea behind multiresolution fusion methods is to extract from the PAN image, using multiresolution techniques, the spatial details not present in LRMS image to inject them into the latter.

Lately, shearlets were introduced with the expressed intent of providing a highly efficient representation of images with edges. In fact, the elements of the shearlet representation form a collection of well-localized waveforms, ranging at various locations, scales and orientations, and with highly anisotropic shapes. This makes the shearlet representation particularly well adapted for representing the edges and other anisotropic objects which are the dominant features in typical images. Therefore, shearlet is well suited for extracting spatial detail information from an image. In addition, it provides a simplified mathematical structure and added flexibility which is particularly useful in image pansharpening [7].

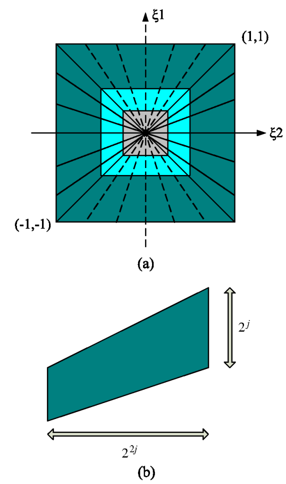


Fig. 1. *The structure of the frequency partition by SIST. (a) The orientation of the frequency plane induced by SIST. (b) The size of the frequency support of SIST.*

The shift-invariant shearlet transform (SIST) [8] is computed as a non-subsampled pyramid filter scheme, that achieves multi-scale partition, followed by directional localization, where the frequency plane is decomposed into a low-frequency subband and 2^l high-frequency subbands, supported on a pair of trapezoids of approximate size $2^{2j} \times 2^j$, oriented along lines of slope $l2^{-j}$, by the shift-invariant shearing filters [9], as shown in Fig. 1. Both stages of the SIST are constructed to be invertible, in order to have a overall invertible system.

Shearlets based fusion methods use the SIST transform to extract the details from the PAN image that are injected into the MS image using several methods as substitution (the simplest one), addition, and other more complex mathematical models. In this paper we cast the algorithm in [10] to use the SIST transform and, hence, we propose a general shearlet based Bayesian fusion method that comprises, as particular cases, substitution, addition and some other popular models.

The paper is organized as follows. In section II the general algorithm for SIST pansharpening is described and the used notation is introduced. Section III models the fusion problem into the Bayesian formulation and inference is performed to obtain the HRMS image. Experimental results and comparison with other methods are presented in IV for synthetic and SPOT5 images and, finally, section V concludes the paper.

II. General SIST-based Pansharpening Algorithm

As already commented, pansharpening methods based on multiresolution decompositions, as SIST, inject the details extracted from the PAN image into the MS image using different models that, following [10], can be classified as

A. *Substitution Model:* It involves completely replacing the MS image details with those of the PAN image.

B. *Additive Model:* It adds the detail information of the PAN image to the detail information of the MS image.

C. *Mathematical-based Model:* It applies a mathematical model to the details information in both PAN and MS images and uses it to weight the information of both images in order to prevent color distortions.

All those models start from the observed low resolution MS image, $\mathbf{Y}$, with B bands, $\mathbf{Y}_i, i = 1, \ldots, B$, each of size $P = M \times N$ pixels, and the PAN image, $\mathbf{x}$, of size $p = m \times n$, with $M < m$ and $N < n$. Based on those observations, pansharpening methods find an estimation of $\mathbf{y}$, the HRMS image, with B bands, $\mathbf{y}_i, i = 1, \ldots, B$, each of size $p = m \times n$ pixels. Based on the algorithm proposed in [10], the following general pansharpening algorithm based on SIST that obtain $\hat{\mathbf{y}}$, an estimation of $\mathbf{y}$, is proposed.

Algorithm 1 *General SIST pansharpening algorithm*

1. Upsample each band of the LRMS image, $\mathbf{Y}_i$, to the size of the PAN image, $\mathbf{x}$, and register the PAN and upsampled MS bands to obtain $\mathbf{s}_i$, $i = 1, \ldots, B$.
2. Apply the SIST decomposition to the PAN image $\mathbf{x}$ and the registered MS image bands $\{\mathbf{s}_i\}$, to obtain the residual (low pass filtered version) SIST coefficient band $\mathbf{x}^R$ and $\{\mathbf{s}_i^R\}$, for the PAN and MS images, respectively, and the SIST detail coefficient bands $\mathbf{x}^{ld}$ and $\{\mathbf{s}_i^{ld}\}$, where the superscript ld indicates the detail bands, with $l = 1, \ldots, L$, representing the scale and $d = 1, \ldots, D$, representing the direction of each coefficient band.
3. Keep the residual band unchanged and merge the details of the PAN $\{\mathbf{x}^{ld}\}$ and MS $\{\mathbf{s}_i^{ld}\}$ images to obtain $\{\hat{\mathbf{y}}_i^{ld}\}$, the reconstructed details of the HRMS image band i at direction ld, as

$$\hat{\mathbf{y}}_i^R = \mathbf{s}_i^R. \tag{1}$$

$$\hat{\mathbf{y}}_i^{ld} = a^{ld}\mathbf{x}^{ld} + b^{ld}\mathbf{s}_i^{ld}, \tag{2}$$

4. Apply the inverse SIST to $\{\hat{\mathbf{y}}_i^R\}$ and $\{\hat{\mathbf{y}}_i^{ld}\}$, to obtain the reconstructed HRMS bands $\{\hat{\mathbf{y}}_i\}$.

Note that the coefficients a^{ld} and b^{ld} control the model for injecting the details of the PAN image into the MS image. Thus, if $b^{ld} = 0$, we obtain the substitution model, and for $a^{ld} = b^{ld} = 1$, we have the additive one. Using different values for a^{ld} and b^{ld} we will get different weighted models proposed in the literature [8], [11]–[14]. Note however, that the models that add the details of both images will also add the noise, contained in the high frequency bands, and the pansharpened image may present a noisy aspect.

In this paper we propose to modify the merging strategy in step 3 of Algorithm 1 by using Bayesian inference as a mathematical way to estimate the detail SIST coefficients of the HRMS image from PAN and LRMS images, providing a sound way to control the noise, preventing color bleeding and generalizing the described previous models.

III. Bayesian Modeling and Inference

Since the PAN image contains the details of the HRMS image but lacks of its spectral information, and the LRMS image have the spectral information of the HRMS images, the relationship between the HRMS band coefficients and those of the PAN and MS images is defined in this paper as,

$$\mathbf{s}_i^R = \mathbf{y}_i^R, \tag{3}$$

$$\mathbf{s}_i^{ld} = \mathbf{y}_i^{ld} + \mathbf{n}_{\mathbf{s}_i}^{ld}, \tag{4}$$

$$\mathbf{x}^{ld} = \mathbf{y}_i^{ld} + \mathbf{n}_{\mathbf{x}}^{ld}, \tag{5}$$

where $\mathbf{n}_{\mathbf{s}_i}^{ld}$ is the noise of coefficients bands, that is assumed to be Gaussian with zero mean and known variances $(\beta_i^{ld})^{-1}$, and $\mathbf{n}_{\mathbf{x}}^{ld}$ is the noise for PAN image of the coefficients bands at each SIST decomposition level, l, and direction, d, that is assumed to be Gaussian with zero mean and known variance $(\gamma_i^{ld})^{-1}$. Note that we are modeling the details band of the PAN image as containing the details of each HRMS band and, hence, the residual band of the PAN image, $\mathbf{x}^R$, will contains the information that is common to all the spectral bands, that is, the non-spectral information. This information is not useful to recover the HRMS spectral bands and it is discarded.

Bayesian methods start with the definition of a prior model where we incorporate the expected characteristics of the original SIST coefficients. For the coefficient band, we choose the Total Variation (TV) prior, that prefers solutions having smooth areas with sharp edges such as the detail bands of the SIST, given by

$$p(\mathbf{y}_i^{ld}) \propto (\alpha_i^{ld})^{p/2} \exp\left\{-\alpha_i^{ld} TV(\mathbf{y}_i^{ld})\right\}, \tag{6}$$

with $TV(\mathbf{y}_i^{ld}) = \sum_{j=1}^{p} \sqrt{(\Delta_j^h(\mathbf{y}_i^{ld}))^2 + (\Delta_j^v(\mathbf{y}_i^{ld}))^2}$ where $\Delta_j^h(\mathbf{y}_i^{ld})$ and $\Delta_j^v(\mathbf{y}_i^{ld})$ represent the horizontal and vertical first order differences at pixel j, respectively, and α_i^{ld} is the parameter model of the MS band i coefficients at level l and direction d. Notice that TV in conjunction to shearlets has been used for denoising [15] with excellent results.

Bayesian methods also need of a degradation model, where fidelity to the observed data is incorporated, expressed as the conditional distribution of the observation given the real data.

From the observation model of the MS image in Eq. (4), we have the following probability distribution

$$p(\mathbf{s}_i^{ld}|\mathbf{y}_i^{ld}) \propto (\beta_i^{ld})^{p/2} \exp\left\{-\frac{1}{2}\beta_i^{ld}\left\|\mathbf{s}_i^{ld}-\mathbf{y}_i^{ld}\right\|^2\right\}. \quad (7)$$

The probability distribution of the details of the PAN image, given HRMS coefficients, from Eq. (5), is written as

$$p(\mathbf{x}^{ld}|\mathbf{y}_i^{ld}) \propto (\gamma_i^{ld})^{p/2} \exp\left\{-\frac{1}{2}\gamma_i^{ld}\left\|\mathbf{x}^{ld}-\mathbf{y}_i^{ld}\right\|^2\right\}. \quad (8)$$

Having defined the degradation and prior models, Bayesian inference is performed based on the posterior distribution of the HRMS image given the observations

$$p(\mathbf{y}_i^{ld}|\mathbf{s}_i^{ld},\mathbf{x}^{ld}) = p(\mathbf{y}_i^{ld})p(\mathbf{s}_i^{ld}|\mathbf{y}_i^{ld})p(\mathbf{x}^{ld}|\mathbf{y}_i^{ld})/p(\mathbf{s}_i^{ld},\mathbf{x}^{ld}) \quad (9)$$

where we are assuming that $\mathbf{s}_i^{ld}$ and $\mathbf{x}^{ld}$ are independent for a given $\mathbf{y}_i^{ld}$.

In order to perform inference from the posterior distribution we need to calculate $p(\mathbf{s}_i^{ld},\mathbf{x}^{ld})$ in Eq. (9). However, $p(\mathbf{s}_i^{ld},\mathbf{x}^{ld})$ cannot be calculated analytically and we will apply the Bayesian variational methodology [16] to approximate the posterior distribution by another one, $q(\mathbf{y}_i^{ld})$, that minimizes the Kullback-Leibler(KL) divergence between $q(\mathbf{y}_i^{ld})$ and $p(\mathbf{y}_i^{ld}|\mathbf{s}_i^{ld},\mathbf{x}^{ld})$.

Due to the use of the TV prior it is also necessary to perform a majorization-minimization procedure, as in [16], to convert the non-quadratic problem to a quadratic one by the introduction of a new parameter $\mathbf{u}_i^{ld}$ that also needs to be estimated with the image. For an estimate of the HRMS image band i in iteration k, $(\hat{\mathbf{y}}_i^{d})^k$, the value of the parameter $\mathbf{u}_i^{ld}$ in iteration $k+1$, $(\mathbf{u}_i^{ld})^{k+1}$, is computed, at each pixel $j=1,\ldots,p$, as

$$(\mathbf{u}_i^{ld})^{k+1}(j) = (\Delta_j^h((\hat{\mathbf{y}}_i^{ld})^k)^2 + (\Delta_j^v((\hat{\mathbf{y}}_i^{ld})^k)^2. \quad (10)$$

With these values, we can obtain a new HRMS image estimate, $(\hat{\mathbf{y}}_i^{d})^k$, the mean of the distribution $q(\mathbf{y}_i^{ld})$ at iteration k, that is defined as

$$(\hat{\mathbf{y}}_i^{ld})^k = (\Sigma_i^{ld})^k\left[\beta_i^{ld}\mathbf{s}_i^{ld} + \gamma^{ld}\mathbf{x}_i^{ld}\right], \quad (11)$$

$$(\Sigma_i^{ld})^k = \left[\alpha_i^{ld}\varsigma(\mathbf{u}_i^{ld})^k + \beta_i^{ld}I_p + \gamma_i^{ld}I_p\right]^{-1}, \quad (12)$$

where

$$\varsigma(\mathbf{u}_i^{ld})^k = (\Delta^h)^t W(\mathbf{u}_i^{ld})^k(\Delta^h) + (\Delta^v)^t W(\mathbf{u}_i^{ld})^k(\Delta^v), \quad (13)$$

with Δ^h and Δ^v represent $p\times p$ convolution matrices associated with the first order horizontal and vertical differences, respectively, and

$$W(\mathbf{u}_i^{ld})^k = diag\left((\mathbf{u}_i^{ld})^k(j)^{-\frac{1}{2}}\right), \quad (14)$$

is a spatial adaptivity matrix that controls the smoothness at each pixel location depending on the strength of the intensity variation at the pixel, expressed as the horizontal and vertical intensity gradient [16].

Returning to the step 3 in the pansharpening Algorithm 1 we obtain the coefficient bands for the estimated HRMS image by replacing Eq. (2) by the result of Eq. (11). Moreover, by setting $\alpha_i^{ld}=\beta_i^{ld}=0$ in Eq. (12) we obtain the SIST-based substitution model. Also, by setting $\alpha_i^{ld}=0$, $\gamma_i^{ld}=\beta_i^{ld}=1$, we get the additive model, while setting γ_i^{ld} and β_i^{ld} to different values generates different mathematical models. Note that in all those cases, α_i^{ld} is set to zero and, hence, no noise control is performed. By setting α_i^{ld} to a non-zero value will help our model to control the noise, while merging the coefficients in the different levels and directions.

TABLE I
SYNTHETIC IMAGE QUANTATIVE RESULTS

Measure	Band	Additive SIST [7]	SR-contourlet [10]	Proposed
COR	b1	0.94	0.99	**0.99**
	b2	0.92	0.99	**0.99**
	b3	0.94	0.98	**0.99**
SSIM	b1	0.71	0.97	**0.98**
	b2	0.84	0.97	**0.97**
	b3	0.79	0.96	**0.98**
PSNR	b1	26.36	38.17	**39.23**
	b2	26.36	39.51	**39.15**
	b3	26.54	36.93	**38.90**
ERGAS	-	8.22	1.61	**1.33**

IV. EXPERIMENTAL RESULTS

In this section, the performance of the proposed SIST-based pansharpening method using the Bayesian inference is evaluated. Results are presented on a synthetic color image and a real SPOT5 image. The observations of the synthetic multispectral image are obtained from the color image, displayed in Fig. 2(a), by convolving it with mask $0.25\times 1_{2\times 2}$ to simulate sensor integration, and then downsampling it by a factor of two by discarding every other pixel in each direction. Zero mean Gaussian noise with variance 16 is then added to obtain the observed MS image shown in Fig. 2(b). For the PAN image, depicted in Fig. 2(c), we used the luminance of the original color image and zero mean Gaussian noise of variance 9 was added. The observed MS image was upsampled to the same size of PAN image by bicubic interpolation and then 4 levels of SIST decomposition was applied on each upsampled MS band and PAN image with 10 and 18 directional levels, for the first two and the last two decomposition levels, respectively. The proposed algorithm was run on those coefficients bands until the criterion $\|(\hat{\mathbf{y}}_i^{ld})^k-(\hat{\mathbf{y}}_i^{ld})^{k-1}\|^2/\|(\hat{\mathbf{y}}_i^{ld})^{k-1}\|^2 < 10^{-4}$ was satisfied, which typically is reached within 3 iterations in the first two decomposition levels, and 4 iterations in the other levels. The values of parameters were experimentally chosen to be $\alpha_i=0.045$, $\beta_b=1/16$, $i=1,2,3$ and $\gamma=0.9$. We compared the proposed SIST using Bayesian inference method with the contourlet-based SR method in [10] and the additive SIST method [7]. The obtained images for each method are displayed in Fig. 2(d)-(f).

To assess spatial improvement of the pansharpened images we use the correlation of the high frequency components (COR) which takes values between zero and one (the higher the value the better the quality of the pansharpened image). Spectral fidelity was assessed by means of the the peak

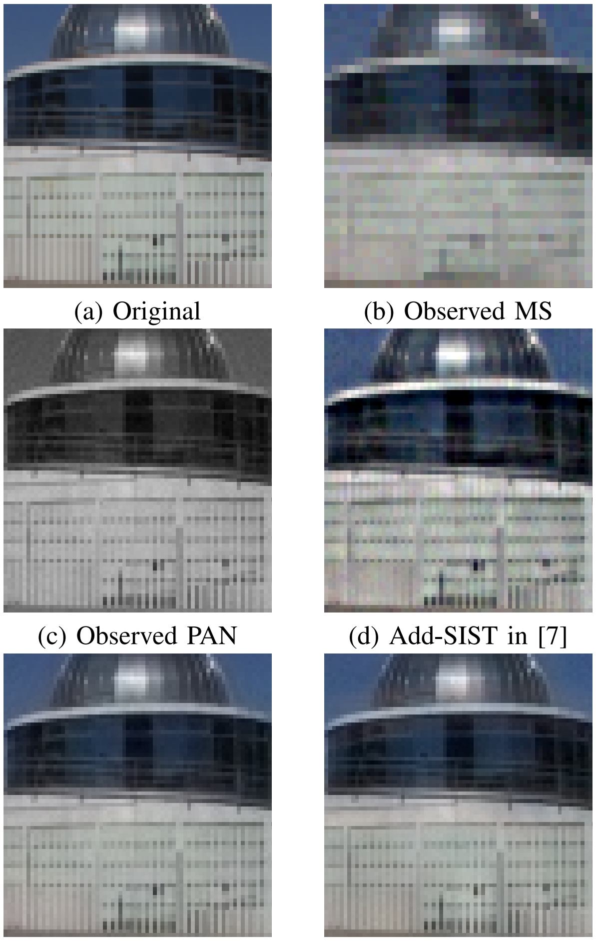

(a) Original (b) Observed MS
(c) Observed PAN (d) Add-SIST in [7]
(e) SR in [10] (f) Proposed method.

Fig. 2. Results for the synthetic image

signal-to-noise ratio (PSNR), the Structural Similarity Index Measure (SSIM), an index ranging from -1 to $+1$, with $+1$ corresponding to exactly equal images, and the ERGAS index, a global criterion for what the lower the value, the higher the quality of the pansharpened image [6]. Table I shows the corresponding quantitative results. The proposed method provides better results for each measure. The COR values reflect that all methods are able to incorporate the details of the PAN image into the pansharpened one. Note the remarkably high values obtained by the contourlet-based SR [10] and the proposed method. The spectral fidelity measures show that the proposed method performs better than the competing methods. The PSNR for the proposed method is about $10dB$ higher than for additive SIST method in [7] with a very high SSIM and low ERGAS values which reflect the high quality of the resulting images. The additive SIST method in [7] incorporates details in all the bands but, since it adds all the high frequency components from the PAN, produces a noisy image (see Fig. 2(d)). The proposed method also provides higher PSNR and SSIM values and a lower ERGAS than the method in [10]. Both the contourlet-based SR [10] and the proposed method (Figs. 2(e) and 2(f), respectively) are able to incorporate details in all the bands while controlling the noise and avoiding color bleeding. However, the proposed method is able to better discriminate noise from high frequency details and, hence, the noise in the image is slightly lower.

In the second experiment, the method was tested on a real SPOT5 dataset. Figure 3(a) shows a region of the RGB color image representing bands 1 to 3 of the MS image. Its corresponding PAN image is depicted in Figure 3(b). Visual inspection of the resulting images, displayed in Figs. 3(c)-(e), reveals similar conclusions to the obtained for the synthetic image. The additive SIST method in [7] (Fig. 3(c)) incorporates most of the details of the PAN image but it is not as sharp as the reconstruction obtained by the method in [10] and the proposed method. These two methods, the contourlet-based SR method in [10] and the proposed one, results in similar images, with a very high spatial and spectral quality although the proposed method is able to better incorporate small details in smooth areas (see the upper left part of the image).

V. Conclusion

In this paper, a new pansharpening method based on the shift-invariant shearlet transform and Bayesian inference has been presented. The proposed method, that has as particular cases substitution, additive, and weighted shearlet fusion strategies, incorporates a solid way to incorporate the details of the PAN into the MS image, while controlling the noise

The efficiency of pansharpening methods has been evaluated by means of visual and quantitative analysis, for synthetic and real data. The proposed method preserves the spectral properties of MS image while incorporating the high frequencies from the PAN image and controlling the noise in the image. Based on the presented experiments, the proposed method significantly outperforms additive-SIST and contourlet-based super-resolution methods.

References

[1] I. Amro, J. Mateos, M. Vega, R. Molina, and A.K. Katsaggelos. A survey of classical methods and new trends in pansharpening of multispectral images. *EURASIP Journal on Advances in Signal Processing*, page 2011:79, September 2011.

[2] B. Aiazzi, L. Alparone, S. Baronti, I. Pippi, and M. Selva. Generalised Laplacian pyramid-based fusion of MS + P image data with spectral distortion minimisation. *ISPRS Internat. Archives Photogramm. Remote Sensing*, 34:36, 2002.

[3] M. González-Audícana and X. Otazu. Comparison between Mallat's and the à trous discrete wavelet transform based algorithms for the fusion of multispectral and panchromatic images. *Int. J. of Remote Sensing*, 26(3):595–614, 2005.

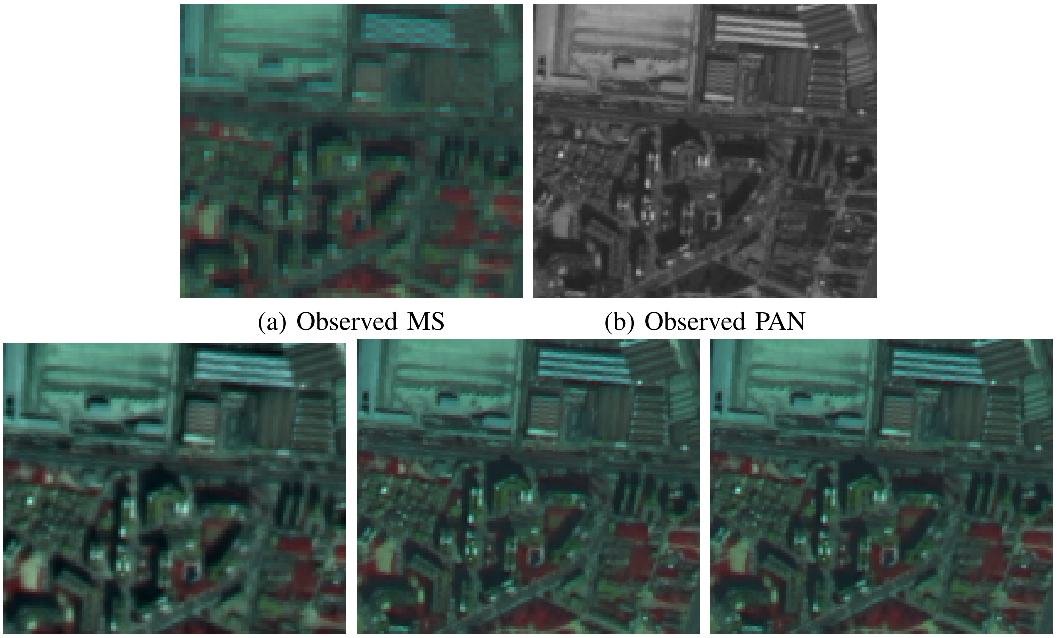
[4] A. L. da Cunha, J. Zhou, and M. N. Do. The nonsubsampled contourlet transform: Theory, design, and applications. *IEEE Trans. on Image Processing*, 15(10):3089–3101, 2006.

[5] M. Lillo-Saavedra and C. Gonzalo. Multispectral images fusion by a joint multidirectional and multiresolution representation. *Int. J. Remote Sens.*, 28:4065 – 4079, 2007.

[6] I. Amro and J. Mateos. *Multispectral Image fusion Using Multiscale and Super-resolution Methods*. VDM Verlag, 2011.

[7] C. Deng, S. Wang, and X. Chen. Remote sensing images fusion algorithm based on shearlet transform. In *Int. Conf. on Environmental Science and Information Application Technology*, volume 3, pages 451–454, 2009.

[8] L. Wang, B. L., and L.-F. Tian. Multi-modal medical image fusion using the inter-scale and intra-scale dependencies between image shift-invariant shearlet coefficients. *Information Fusion*, 2012, in press.



(a) Observed MS (b) Observed PAN

(c)Add-SIST in [7] (d) SR in [10] (e) proposed method.

Fig. 3. Results for the SPOT5 image

[9] G. Easley, D. Labate, and W.-Q. Lim. Sparse directional image representations using the discrete shearlet transform. *Applied and Computational Harmonic Analysis*, 25(1):25–46, 2008.

[10] I. Amro, J. Mateos, and M. Vega. General contourlet pansharpening method using Bayesian inference. In *2010 European Signal Processing Conference (EUSIPCO-2010)*, pages 294–298, August 2010.

[11] Z.-H. Wang, J.-Q. Wang, D.-G. Zhao, and W. Fu. Image fusion based on shearlet and improved PCNN. *Laser & Infrared*, 2:023, 2012.

[12] Q. Miao, C. Shi, P. Xu, M. Yang, and Y. Shi. Multi-focus image fusion algorithm based on shearlets. *Chinese Optics Letters*, 9(4):041001, 2011.

[13] Q. Miao, C. Shi, P. Xu, M. Yang, and Y. Shi. A novel algorithm of image fusion using shearlets. *Optics Communications*, 284(6):1540–1547, 2011.

[14] L. Lu, J. Zhao, and H. Sun. Multi-focus image fusion based on shearlet and local energy. In *2nd Int. Conf. on Signal Processing Systems*, volume 1, page 632, 2010.

[15] G.R. Easley, D. Labate, and F. Colonna. Shearlet-based total variation diffusion for denoising. *IEEE Trans. on Image Processing*, 18(2):260–268, 2009.

[16] M. Vega, J. Mateos, R. Molina, and A.K.. Katsaggelos. Super resolution of multispectral images using TV image models. In *2th Int. Conf. on Knowledge-Based and Intelligent Information & Eng. Sys.*, pages 408–415, 2008.