PARAMETER ESTIMATION IN MULTICHANNEL IMAGE RESTORATION. APPLICATIONS IN ASTRONOMY *

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Abstract

In this paper new and previously developed methods to restore images corrupted with Gaussian noise are applied to multichannel astronomical images. New methods to estimate the parameters appearing in multichannel restoration are proposed, they are finally tested on real astronomical images.

Keywords: Multichannel Image restoration

1 Introduction

The use of image data from multiple frequency bands, multiple time frames, or multiple sensors can be of tremendous value in a number of applications, such as multispectral satellite remote sensing, multisensor robot guidance, multimedium medical diagnosis and obviously astronomical image restoration.

Multichannel image processing differs from single channel image processing because of the redundancy and the complementary feature of information within channels. The processing is much more complicated due to the increased dimension and the need for extracting and exchanging information from and among all channels.

There has been an enormous interest on the multichannel (multiband) image restoration problem (see, for example, [1, 2, 7] and its references).

In this paper we will study the application of multichannel Bayesian restoration techniques to astronomical images. The paper is organized as follows. In section 2 we introduce the Bayesian paradigm with its corresponding image and noise models. Using these image and noise models we propose a multichannel image restoration method, described in section 3. The estimation of the model parameters is described in section 4. Finally, section 5 discusses the results obtained by the application of our restoration method to astronomical multichannel images.

*This work has been supported by the "Comisión Nacional de Ciencia y Tecnología" under TIC-989. project: "Investigación y Desarrollo de Software para la restauración y compresión bayesiana de imágenes. Aplicaciones a medicina nuclear y astronomía".

2 The Multichannel Restoration Problem

Let f be the original multichannel image which would be observed under ideal conditions (i.e. no noise and no distortions produced by blurring and instrumental effects), composed of the f_i images of the nQ channels with size $M \times N$, that is,

$$f = (f_0^t, f_1^t, ..., f_{nQ-1}^t)^t. \tag{1}$$

The aim is to reconstruct the multichannel image f from the observed image g that we express in terms of the nQ observed channel images g_i as in Eq. (1).

Bayesian methods start with a *prior distribution*, $P(f)$, a probability distribution over images f. It is here where we incorporate information on the expected structure within an image. It is also necessary to specify $P(g/f)$, the probability distribution of observed images g if f were the 'true' image. In this framework, the estimation of the original image, $\hat{f}$, is usually obtained by

$$\hat{f} = \arg\min_f \{-\log[P(f) \times P(g/f)]\}. \tag{2}$$

Let us now examine the degradation, $P(g/f)$, and image, $P(f)$, models.

2.1 Degradation Model

The observed image g is degraded by blurring and noise. Thus, the degradation model we consider here is

$$g = Hf + \epsilon \tag{3}$$

where f and g represents the original and observed image respectively, ϵ is an additive Gaussian noise, $\epsilon \sim \mathcal{N}(0, K_n)$, H is the multichannel atmospheric blur matrix that does not mix signal across channels and K_n is the multichannel Gaussian noise variance matrix $\left(K_{n_0}, K_{n_1}, \ldots, K_{n_{nQ-1}}\right)$.

So we have

$$P(g/f) \propto \exp\left[-\sum_{i=0}^{nQ-1} \frac{1}{2K_{n_i}} \parallel (g_i - (Hf)_i) \parallel^2\right], \tag{4}$$

where K_{n_i} is the multichannel noise model variance in channel i.

2.2 Image Models

For the prior model is usual to choose Gaussian probability distributions, which in the case of $P(f)$ constitute Gibbs priors, but with an added term $-\frac{1}{2}f^t\Gamma f$, which takes into account the between channels correlations, of the form

$$f^t\Gamma f = \sum_{0 \leq iQ < jQ < nQ} \gamma_{iQ,jQ} \left|f_{iQ} - f_{jQ}\right|^2. \tag{5}$$

This means that the image model we are using has the form

$$P(f) \propto \exp\left\{-\frac{1}{2}\sum_{i=0}^{NQ-1}\left(\frac{1}{K_{s_i}} f_i^t (I_{MN} - C) f_i\right) - f^t\Gamma f\right\}, \tag{6}$$

where K_{s_i} is the multichannel prior model variance. In Eq. (6) also appears the CAR model [9] matrix for each channel, $f_{iQ}^t(I_{MN}-C)f_{iQ}$, which measures the similarity of each pixel with its four closest neighbors within each image band.

When now trying to estimate the original image, that is, when we try to solve Eq. (2), we are faced with two problems:

- Technical problems due to the size of the resulting matrices. There exist approaches to overcome this problem [1, 3].
- Model problems: a correlation term as that of Eq. (5) gives more importance to channels with greater fluxes or to its pixel values in a given zone. However, this quantitative flux between channels is not what we are looking for, but a qualitative flux of information between channels.

To overcome the second problem mentioned above the γ factors in Eq. (5) must be corrected by introducing a term like $1/(\| f_{iQ} \| \| f_{jQ} \|)$ [2]. This produces satisfactory results in experiment with photographic images whose blue, red and green channels are artificially distorted with Gaussian noise, however, the method does not keep the flux inside each channel.

Trying to adapt the method to astronomical images we started by replacing the correction terms by one having the form $1/(\Phi_{(iQ)}\Phi_{(jQ)})$ [7], where $\Phi_{(iQ)}$ is the flux of the iQ channel, but that did not work very well, since it reduced excessively the magnitude of the correlation contribution to the prior. However this correction term solves part of the model problems since it gives to the information provided by each channel a weight not directly related to its absolute pixel values. This will allow to highlight image objects that appear more clearly in other channels, even if the pixel value magnitudes of these other channel were lower.

Using a correlation term like the previously proposed, the γ parameter values have to be of a very high order of magnitude to appreciate its effect. It was also observed a dependency on the K_n values used over the order of magnitude necessary. For this reason, we propose a modification of the γ parameter values in Eq. (5) as follows:

$$f^t\Gamma f = \sum_{0\le iQ<jQ<nQ} \frac{\Phi}{\Phi_{iQ}\Phi_{jQ}\left(K_{niQ}K_{njQ}\right)} \left|f_{iQ}-f_{jQ}\right|^2 , \tag{7}$$

where Φ is the mean of the total fluxes of the nQ channels.

To alleviate the flux transfer problem between channels, the evaluation of Eq. (2) will be constrained to conserve the flux of the channels.

3 Algorithm

In order to find an estimation of f, that is, to solve Eq. (2), we have to take into account the noise model $P(g/f)$ in Eq. (4) and the prior model, $P(f)$, in Eq. (6) with Γ defined by Eq. (7).

Then, the solution to Eq. (2) can be found by solving

$$\hat{f} = \arg\min_f \left\{ \sum_{i=0}^{nQ-1} \frac{1}{2K_{n_i}} \| g_i - (Hf)_i \|^2 + \frac{1}{2}\sum_{i=0}^{nQ-1} \left(\frac{1}{K_{s_i}} f_i^t (I_{MN}-C) f_i \right) + f^t\Gamma f \right\}. \tag{8}$$

Equation (8) is solved with flux conservation constraint $\Phi_{iQ} = \psi_{iQ}$ for all iQ values, where Φ_{iQ} and ψ_{iQ} are the flux of the iQ band of the solution f and the original image, respectively. We will take the approach that

$$\psi_{iQ} = 1_{MN}^t \hat{f}_{iQ} \cong 1_{MN}^t g_{iQ} \tag{9}$$

assuming that $1_{MN}^t \epsilon_{iQ} \cong 0$.

With the techniques described in [3], Eq. (8) can be solved in the Fourier domain and, in this domain, the flux conservation constraint can be easily applied noting that

$$\Phi_{iQ} = MN \left(\mathcal{F}\left[f_{iQ}\right]\right)_{(0)} , \tag{10}$$

where $\mathcal{F}[f_{iQ}]$ is the Fourier transform of f_{iQ}. Thus, flux conservation implies constraining the value of the first component of the Fourier transform of f_{iQ}.

4 Parameter Estimation

It is clear that in order to solve Eq. (8), that is, in order to obtain $\hat{f}$, we have to determine the value of the parameters K_n and K_s.

For the estimation of the noise variances K_n, we have used the method proposed in [4]. This method is based on searching small "homogeneus" regions in the images, where it is assumed that the sample variance is due only to the noise. The method operates in two steps:

1. First, it divides the image in squared regions of size $2^l \times 2^l$, $l = 3, \ldots$, in a way that is frequently used in multiresolution algorithms. The fours smallest sample variance values, for each level, are combined using an outlier analysis into one estimate of the smallest variance in each level. The result is a sequence of variance estimates q_l for $l = 3, \ldots$.

   An important property of the noise variance estimation is that its value increases monotonically with the size of the sample. This is a consequence of the well known fact that the variance is a consistent estimator, its confidence interval decreases with the increase of degrees of freedom of the sample. This behavior is analyzed in detail in [4].

2. In a second step, K_n is obtained through an analysis of the sequence q_l, by comparing the rate of growing of the sequence of experimental variance estimations with the theorical grow for a model of pure Gaussian noise.

The method does not use any segmentation or edge-detection algorithms to separate noise and signal regions, so it is robust and well suited to be used in astronomical images.

Knowing K_n, it is possible to determine empirically the signal to noise ratio and derive from it a plausible value for our prior model variance K_s.

5 Experimental Results

The proposed algorithm has been tested on the multichannel astronomical image shown in Fig. 1(a)–(c), corresponding to the same object taken at different wavelength. Note that all the bands are shown in logarithmic scale. The flux of each band is 17059 for Fig. 1(a), 83087 for Fig. 1(b) and 37820 for Fig. 1(c). Note that the range for each image is $[0, 10]$ for figure 1(a), $[0, 62]$ for figure 1(b) and $[0, 24]$ for figure 1(c).

The atmospherical blurring, H, can be approximated by

$$H_{iQ}(r) \propto \left[1 + \left(\frac{r}{R_{iQ}}\right)^2\right]^{-\beta}, \tag{11}$$

with the condition that for every channel $H_{iQ} 1_{MN} = 1_{MN}$, 1^t_{MN} a MN-dimensional vector with values $(1, 1, 1, \ldots, 1)$. We found $\beta \sim 3$ and $R \sim 3.4$ pixels in all the channels.

Applying the proposed parameter estimation technique we obtained that $K_n = (378 \times 10^{-6}, 1318 \times 10^{-6}, 1202 \times 10^{-6})$ being the obtained signal to noise ratio for each band 28.61dB, 38.82dB and 30.65dB, respectively.

Figure 1(d) shows the result of applying a monochannel restoration scheme, supposing Gaussian noise, on the image shown in Fig. 1(a). In Fig. 1(e) the result of applying the multichannel Gaussian filter proposed.

The analysis of the results shows that the multichannel approximation clearly improves the image while the monochannel restoration presents more noise. As an example of the qualitative flux of information between images one can observe that the two stars in the upper left corner of the image in Fig. 1(a) almost disappeared in the monochannel restoration while they are preserved and enhanced by our proposed multichannel algorithm.

Acknowledgements

We would like to thank A. del Olmo and J. Perea, members of the Instituto de Astrofísica de Andalucia, for providing us with the images we have used in this paper.

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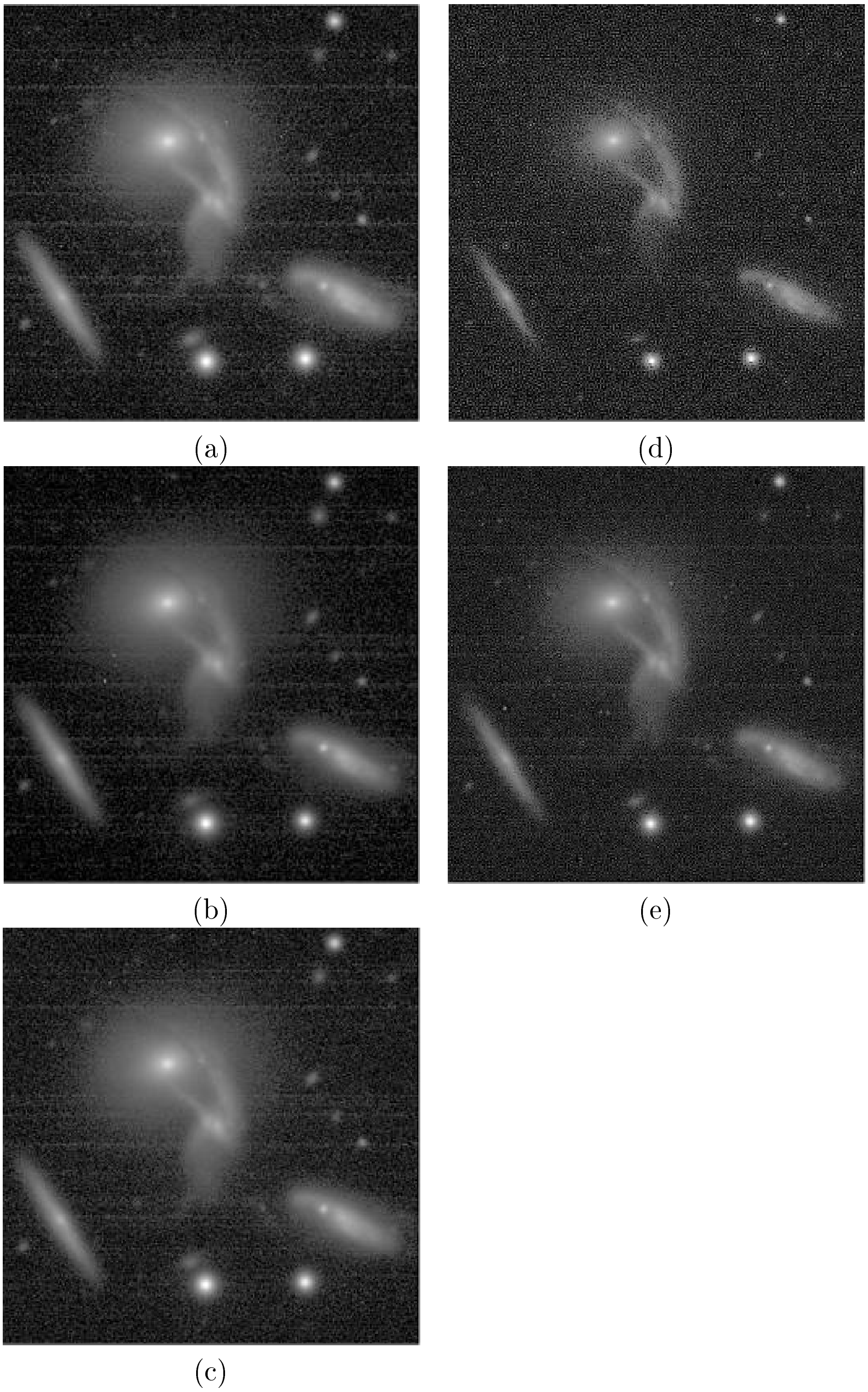

Figure 1: *Observed image at three different wavelength corresponds to (a), (b) and (c). (d) Restoration of (a) using a monochannel algorithm and (e) Restoration of (a) using the proposed multichannel algorithm.*

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