Bayesian Reconstruction of BDCT Compressed Images

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Abstract

High compression ratios for both still images and sequences of images are usually achieved by discarding information represented by block discrete cosine transform (BDCT) coefficients which is considered unimportant. This compression procedure yields images that exhibit annoying blocking artifacts. In this paper, we propose a method based on a hierarchical Bayesian approach for the reconstruction of BDCT compressed images and the estimation of the related parameters which results in the removal of the blocking artifact.

Key Words : Reconstruction, Post-processing, image coding,

1 INTRODUCTION

Block-transform coding is by far the most popular approach for image compression. Evidence of this fact is that both the Joint Photograph Experts Group (JPEG) and the Motion Pictures Experts Group (MPEG) recommend the use of the block discrete cosine transform (BDCT) for compressing still and sequences of motion images, respectively [1]. However, such a compression method has the problem of the existence of blocking artifacts in high compressed ratio images.

This artifact manifests itself as an artificial discontinuity between adjacent blocks. It is a direct result of the independent processing of the blocks which does not take into account the between-block pixel correlations. It constitutes a serious bottleneck for many important visual communication applications that require visually pleasing images at very high compression ratios. The recent progress in VLSI technology makes us believe that the incorporation of recovery algorithms at the decoders is a very promising approach to bridging the conflicting requirements of high-quality images and high compression ratios. In the past various algorithms have been proposed to improve quality of block-transform compressed images in the decoder without increasing the bit-rate. In the JPEG standard [1] a technique for predicting the AC coefficients is recommended in Annex-K.8.2. However, in areas with sharp intensity transitions such a prediction scheme fails. In [8, 9], methods to reconstruct the image using a spatially-invariant prior knowledge about the original image along with the transmitted data are proposed. Another MAP approach based on a compound Gauss Markov image model and mean field annealing was proposed in [7]. In [10], Yang, *et al.* proposed a POCS based recovery algorithm that uses a spatially-adaptive constraint that enforces between-block smoothness.

The method proposed in [10] requires the estimation of the regularization parameter. In this paper we formulate their algorithm within the Bayesian paradigm and perform the estimation of the regularization parameters, representing the image and noise variances, following the hierarchical Bayesian approach [4, 5, 6]. We show how, following this approach, it is possible to incorporate knowledge about the unknown hyperparameters ranging from vague to very precise, into the algorithm.

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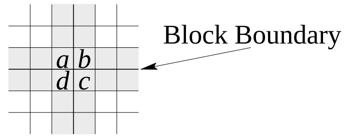

Figure 1: Position of the pixels a, b, c, d in a 4 block intersection.

The rest of the paper is organized as follows. In section 2 the mathematical background for this paper is established. Section 3 describes the prior and noise models for the reconstruction problem. In section 4 we describe the use of the hierarchical approach to the hyperparameter estimation and the recovery algorithm is presented. Experimental results are presented in section 5 and, finally, section 6 concludes the paper.

2 MATHEMATICAL DEFINITION OF THE RECOVERY PROBLEM

Throughout this paper a digital $N \times N$ image is treated as a $N^2 \times 1$ vector in the R^{N^2} space by lexicographic ordering either the rows or the columns. The BDCT is viewed as a linear transformation from R^{N^2} to R^{N^2}. Then, for an image $\mathbf{f}$ we can write $\mathbf{F} = \mathbf{Bf}$, where $\mathbf{F}$ is the BDCT of $\mathbf{f}$ and $\mathbf{B}$ is the BDCT matrix. To achieve a bit-rate reduction for transmission, each element of $\mathbf{F}$ is quantized. This quantization operator can be described mathematically by a mapping or an operator, $\mathcal{Q}$ from R^{N^2} to R^{N^2}. The input-output relation of the coder can be modeled by $\mathbf{G} = \mathcal{Q}\mathbf{Bf}$. Due to the unitary property of the DCT matrices, the BDCT matrix is also unitary and the inverse transform can be simply expressed by $\mathbf{B}^t$ where t denotes the transpose of a matrix. In the receiver only the quantized BDCT coefficients $\mathbf{G}$ are available and the output of a conventional decoder is $\mathbf{g} = \mathbf{B}^t\mathbf{G}$.

The reconstruction problem calls for finding an estimate of $\mathbf{f}$ given $\mathbf{g}$, $\mathcal{Q}$ and, possibly, knowledge about $\mathbf{f}$. In this work we propose to reconstruct the artificial block boundaries, where the blocking artifact is more visible, keeping the rest of the image unchanged. Following the Bayesian paradigm a degradation model is needed which describes how the blocky image is obtained from the real one. In addition, a model describing our knowledge about the original image, the image before coding is also required. Let us study these models in detail.

3 IMAGE AND NOISE MODELS

If we assume that 8×8 blocks are used and the image is 512×512, we can define $\mathbf{u}_c$ and $\mathbf{x}_c$, as a column vector with the elements of $\mathbf{f}$ and $\mathbf{g}$, respectively, that are at the left of a block boundary column but not in a four blocks intersection, that is, $\mathbf{u}_c = \{\mathbf{f}(u)\},\ \mathbf{x}_c = \{\mathbf{g}(u)\},\ u = (x, y),\ x = 8 * i,\ y = 8 * j + l,\ i = 1, 2, \ldots, 64,\ j = 1, 2, \ldots, 64,\ l = 2, 3, \ldots, 7$ and $\mathbf{v}_c$ and $\mathbf{y}_c$, as a column vector with the elements of $\mathbf{f}$ and $\mathbf{g}$, respectively, that are at the right of a block boundary column but not in a four blocks intersection, that is, $\mathbf{v}_c = \{\mathbf{f}(u)\},\ \mathbf{y}_c = \{\mathbf{g}(u)\},\ u = (x, y),\ x = 8 * i + 1,\ y = 8 * j + l,\ i = 1, 2, \ldots, 64,\ j = 1, 2, \ldots, 64,\ l = 2, 3, \ldots, 7$.

In a similar way we define $\mathbf{u}_r$, $\mathbf{v}_r$, $\mathbf{x}_r$ and $\mathbf{y}_r$, the column vectors representing the rows at the block boundaries of $\mathbf{f}$ and $\mathbf{g}$, respectively.

We also define $\mathbf{f}_u(a), \mathbf{f}_u(b), \mathbf{f}_u(c), \mathbf{f}_u(d)$ and $\mathbf{g}_u(a), \mathbf{g}_u(b), \mathbf{g}_u(c), \mathbf{g}_u(d),\ u = (i, j)$ as the pixels in a four blocks boundary of $\mathbf{f}$ and $\mathbf{g}$, respectively, as represented in Fig 1.

In order to capture the vertical local properties of the image we define a $(512 * 63) \times (512 * 63)$

diagonal matrix, $\mathbf{W}_c$, of the form

$$\mathbf{W}_c = \begin{bmatrix} \omega_c(1) & 0 & 0 & \cdots & 0 \\ 0 & \omega_c(2) & 0 & \cdots & \vdots \\ 0 & 0 & \ddots & 0 & \vdots \\ \vdots & \vdots & 0 & \ddots & 0 \\ 0 & \cdots & \cdots & 0 & \omega_c(512*63) \end{bmatrix},$$

where the $\omega_c(i)$'s, $i = 1, 2, \ldots, (512*63)$ will weight each block boundary pixel. Analogously, we can define $\mathbf{W}_r$ to capture the horizontal local properties of the image (see [10, 2] for details).

With the above definitions, the prior knowledge about the smoothness in the block boundaries of the image has the form

$$\begin{aligned} p(\mathbf{f} \mid \alpha) \propto \alpha^{-\frac{p}{2}} \exp\left\{-\frac{1}{2}P(\mathbf{f} \mid \alpha)\right\} = \alpha^{-\frac{p}{2}} \exp\Big\{-\frac{1}{2}\alpha \Big\{2 \parallel \mathbf{W}_c(\mathbf{u}_c - \mathbf{v}_c) \parallel^2 + \\ 2 \parallel \mathbf{W}_r(\mathbf{u}_r - \mathbf{v}_r) \parallel^2 + \parallel \mathbf{W}_c(\mathbf{f}(a) - \mathbf{f}(b)) \parallel^2 + \parallel \mathbf{W}_r(\mathbf{f}(b) - \mathbf{f}(c)) \parallel^2 + \\ \parallel \mathbf{W}_c(\mathbf{f}(c) - \mathbf{f}(d)) \parallel^2 + \parallel \mathbf{W}_r(\mathbf{f}(d) - \mathbf{f}(a)) \parallel^2\Big\}\Big\}, \end{aligned}$$

with $p = 961 \times 63$ and α measure of the roughness between two block boundaries.

Fidelity to the data at these block boundaries is expressed by

$$\begin{aligned} p(\mathbf{g} \mid \mathbf{f}, \beta) \propto \beta^{-r} \exp\left\{-\frac{1}{2}N(\mathbf{g} \mid \mathbf{f}, \beta)\right\} = \\ \beta^{-r} \exp\Big\{-\frac{1}{2}\beta \Big\{\parallel \mathbf{u}_c - \mathbf{x}_c \parallel^2 + \parallel \mathbf{v}_c - \mathbf{y}_c \parallel^2 + \parallel \mathbf{u}_r - \mathbf{x}_r \parallel^2 + \parallel \mathbf{v}_r - \mathbf{y}_r \parallel^2 + \\ \parallel \mathbf{f}(a) - \mathbf{g}(a) \parallel^2 + \parallel \mathbf{f}(b) - \mathbf{g}(b) \parallel^2 + \parallel \mathbf{f}(c) - \mathbf{g}(c) \parallel^2 + \parallel \mathbf{f}(d) - \mathbf{g}(d) \parallel^2 + \Big\}\Big\}, \end{aligned}$$

where β is defined as $\beta^{-1} = \sigma^2_{noise}$, the noise variance of the image, and $r = 898 \times 63$.

If α and β are known, then following the Bayesian paradigm it is customary to select, as the reconstruction of $\mathbf{f}$, the image $\mathbf{f}_{(\alpha,\beta)}$ defined by

$$\mathbf{f}_{(\alpha,\beta)} = \arg\min_{\mathbf{f}} \{M(\mathbf{f}, \mathbf{g} \mid \alpha, \beta)\} = \arg\min_{\mathbf{f}} \{P(\mathbf{f} \mid \alpha) + N(\mathbf{g} \mid \mathbf{f}, \beta)\}. \tag{1}$$

4 PROPOSED ALGORITHM

Given α and β, $\mathbf{f}_{(\alpha,\beta)}$ is estimated by differentiating Eq. (1) with respect to $\mathbf{f}$, obtaining

$$\begin{aligned} \mathbf{u}_c(i) &= 0.5[1 + \beta(\beta + 4\alpha\omega_c^2(i))^{-1}]\mathbf{x}_c(i) + 0.5[1 - \beta(\beta + 4\alpha\omega_c^2(i))^{-1}]\mathbf{y}_c(i) && (2) \\ \mathbf{v}_c(i) &= 0.5[1 - \beta(\beta + 4\alpha\omega_c^2(i))^{-1}]\mathbf{x}_c(i) + 0.5[1 + \beta(\beta + 4\alpha\omega_c^2(i))^{-1}]\mathbf{y}_c(i) && (3) \\ \mathbf{u}_r(i) &= 0.5[1 + \beta(\beta + 4\alpha\omega_r^2(i))^{-1}]\mathbf{x}_r(i) + 0.5[1 - \beta(\beta + 4\alpha\omega_r^2(i))^{-1}]\mathbf{y}_r(i) && (4) \\ \mathbf{v}_r(i) &= 0.5[1 - \beta(\beta + 4\alpha\omega_r^2(i))^{-1}]\mathbf{x}_r(i) + 0.5[1 + \beta(\beta + 4\alpha\omega_r^2(i))^{-1}]\mathbf{y}_r(i), && (5) \end{aligned}$$

and, for pixel in a four block intersection, the solution of the following equation system

$$\begin{aligned} \alpha[\omega_c^2(a)(\mathbf{f}(a) - \mathbf{f}(b)) + \omega_r^2(a)(\mathbf{f}(a) - \mathbf{f}(d))] + \beta(\mathbf{f}(a) - \mathbf{g}(a)) &= 0 \\ \alpha[\omega_c^2(a)(\mathbf{f}(b) - \mathbf{f}(a)) + \omega_r^2(c)(\mathbf{f}(b) - \mathbf{f}(c))] + \beta(\mathbf{f}(b) - \mathbf{g}(b)) &= 0 \\ \alpha[\omega_c^2(c)(\mathbf{f}(c) - \mathbf{f}(d)) + \omega_r^2(c)(\mathbf{f}(c) - \mathbf{f}(b))] + \beta(\mathbf{f}(c) - \mathbf{g}(c)) &= 0 \\ \alpha[\omega_c^2(b)(\mathbf{f}(d) - \mathbf{f}(c)) + \omega_r^2(a)(\mathbf{f}(d) - \mathbf{f}(a))] + \beta(\mathbf{f}(d) - \mathbf{g}(d)) &= 0. \end{aligned} \tag{6}$$

It is obvious that in order to estimate the original image, that is, in order to solve Eq. (1) we need to know or estimate the unknown hyperparameters. Yang *et al.*[9, 10] proposed empirical procedures to estimate these parameters within the POCS and CLS approaches to the reconstruction problem. In this paper we follow the so called hierarchical Bayesian paradigm, a paradigm where prior knowledge about the unknown hyperparameters can be incorporated into the algorithm if available or they can be estimated without any prior knowledge.

This hierarchical paradigm applied to our problem uses the global probability defined as

$$p(\alpha,\beta,\mathbf{f},\mathbf{g}) = p(\alpha)p(\beta)p(\mathbf{f}\mid\alpha)p(\mathbf{g}\mid\mathbf{f},\beta).$$

Let us examine all the term involved in this equation. Having defined in the previous section the image and degradation models, that is, $p(\mathbf{f}\mid\alpha)$ and $p(\mathbf{g}\mid\mathbf{f},\beta)$, we have to specify the probabilities on the hyperparameters.

In situations without prior information what is needed is a non informative prior on the hyperparameters, α and β (the term non informative is meant to imply that no information about the hyperparameters is contained in the priors). For the problem at hand we can use improper non informative priors $p(\omega)\propto const$ over $[0,\infty)$ where ω denotes a hyperparameter. However, it is also possible, to incorporate precise prior knowledge about the value of the noise and prior variances [3, 5]. With the improper hyperpriors, the method proposed amounts to selecting $\hat{\alpha}$ and $\hat{\beta}$, as the maximum likelihood estimates, *mle*, of α and β from $p(\mathbf{g}\mid\alpha,\beta)$.

Let us describe the estimation process in detail. Let us fix α and β and expand $M(\mathbf{f},\mathbf{g}\mid\alpha,\beta)$ around $\mathbf{f}_{(\alpha,\beta)}$. We then have,

$$p(\alpha,\beta\mid\mathbf{g})\propto p(\mathbf{g}\mid\alpha,\beta)\propto\frac{\exp\{-\frac{1}{2}M(\mathbf{f}_{(\alpha,\beta)},\mathbf{g}\mid\alpha,\beta)\}}{\alpha^{\frac{p}{2}}\beta^{r}}\times$$

$$\int_{\mathbf{f}}\exp\left\{-\frac{1}{2}(\mathbf{f}-\mathbf{f}_{(\alpha,\beta)})^{t}\mathbf{Q}(\alpha,\beta)(\mathbf{f}-\mathbf{f}_{(\alpha,\beta)})\right\}d\mathbf{f} = \frac{\exp\{-\frac{1}{2}M(\mathbf{f}_{(\alpha,\beta)},\mathbf{g}\mid\alpha,\beta)\}det\,\mathbf{Q}(\alpha,\beta)^{-\frac{1}{2}}}{\alpha^{\frac{p}{2}}\beta^{r}},$$

where $\mathbf{Q}(\alpha,\beta)$ is a matrix which can be represented as a diagonal matrix with elements, $\mathbf{q}_{i,i}(\alpha,\beta)$, defined as

$$\mathbf{q}_{i,i}(\alpha,\beta) = 2\beta\mathbf{I}+\alpha\mathbf{A}_i = 2\beta\mathbf{I}+\alpha\begin{bmatrix}4\omega_c^2(i) & -4\omega_c^2(i)\\ -4\omega_c^2(i) & 4\omega_c^2(i)\end{bmatrix}$$

for pixel locations $i\in\mathbf{u}_c$

$$\mathbf{q}_{i,i}(\alpha,\beta) = 2\beta\mathbf{I}+\alpha\mathbf{A}_i = 2\beta\mathbf{I}+\alpha\begin{bmatrix}4\omega_r^2(i) & -4\omega_r^2(i)\\ -4\omega_r^2(i) & 4\omega_r^2(i)\end{bmatrix}$$

for pixel locations $i\in\mathbf{u}_r$

$$\begin{aligned}\mathbf{q}_{i,i}(\alpha,\beta) &= 2\beta\mathbf{I}+\alpha\mathbf{A}_i\\ &= 2\beta\mathbf{I}+\alpha\begin{bmatrix}2\omega_c^2(a)+2\omega_r^2(a) & -2\omega_c^2(a) & 0 & -2\omega_r^2(a)\\ -2\omega_c^2(a) & 2\omega_c^2(a)+2\omega_r^2(c) & -2\omega_r^2(c) & 0\\ 0 & -2\omega_r^2(c) & 2\omega_c^2(c)+2\omega_r^2(c) & -2\omega_c^2(c)\\ -2\omega_r^2(a) & 0 & -2\omega_c^2(c) & 2\omega_c^2(c)+2\omega_r^2(a)\end{bmatrix},\end{aligned}$$

for pixel locations i in a four blocks intersection.

Differentiating $-\log p(\alpha,\beta\mid\mathbf{g})$ with respect to α and β we have

$$\begin{aligned}&2\parallel\mathbf{W}_c(\mathbf{u}_c-\mathbf{v}_c)\parallel^2+2\parallel\mathbf{W}_r(\mathbf{u}_r-\mathbf{v}_r)\parallel^2+\parallel\mathbf{W}_c(\mathbf{f}(c)-\mathbf{f}(d))\parallel^2+\parallel\mathbf{W}_r(\mathbf{f}(d)-\mathbf{f}(a))\parallel^2+\\ &\parallel\mathbf{W}_c(\mathbf{f}(a)-\mathbf{f}(b))\parallel^2+\parallel\mathbf{W}_r(\mathbf{f}(b)-\mathbf{f}(c))\parallel^2+\sum tr[\mathbf{q}_{i,i}^{-1}(\alpha,\beta)\mathbf{A}_i] = \frac{p}{\alpha}\end{aligned}\tag{7}$$

$$\begin{aligned}&\parallel\mathbf{u}_c-\mathbf{x}_c\parallel^2+\parallel\mathbf{v}_c-\mathbf{y}_c\parallel^2+\parallel\mathbf{u}_r-\mathbf{x}_r\parallel^2+\parallel\mathbf{v}_r-\mathbf{y}_r\parallel^2+\parallel\mathbf{f}(a)-\mathbf{g}(a)\parallel^2+\\ &\parallel\mathbf{f}(b)-\mathbf{g}(b)\parallel^2+\parallel\mathbf{f}(c)-\mathbf{g}(c)\parallel^2+\parallel\mathbf{f}(d)-\mathbf{g}(d)\parallel^2+\sum tr[\mathbf{q}_{i,i}^{-1}(\alpha,\beta)] = \frac{r}{2\beta}\end{aligned}\tag{8}$$

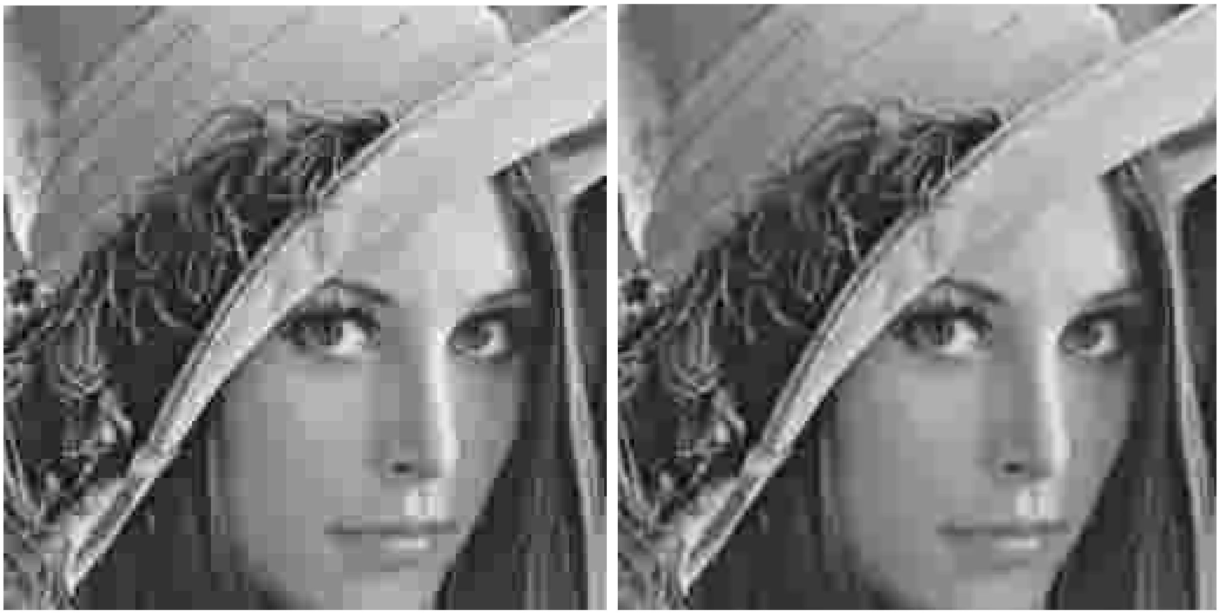

Figure 2: 256×256 center section of blocky "Lena" from JPEG based compression at .24 bpp and its reconstruction

Algorithm	$PSNR$
JPEG Blocky	29.58dB
AC Prediction	29.52dB
POCS Nonadaptive	30.32dB
POCS Adaptive	30.43dB
Proposed Algorithm	30.47dB

Table 1: $PSNR$ of the reconstructed images with different algorithms.

In summary, the following iterative algorithm can be used to recover $\mathbf{f}$, an image with reduced blocking artifact (this method can also be considered as an EM procedure [3]).

1. Set $\mathbf{f}^0 = \mathbf{g}$. Choose α^0 and β^0.
2. For $k = 1, 2, \ldots$
   (a) Estimate α^k and β^k by substituting α^{k-1} and β^{k-1} in the left hand side of Eqs. (7) and (8), respectively.
   (b) For $i = 1, 2, \ldots$, compute $\mathbf{f}^k$ from Eqs. (2), (3), (4), (5) and (6).
3. Goto 2 until $\| \mathbf{f}^k - \mathbf{f}^{k-1} \|$ is less than a prescribed bound.

5 TEST EXAMPLES

In this section, experiments are presented in order to test the proposed recovery algorithm. The 512×512 "Lena" image is used. The image was compressed using a JPEG based coder-decoder at a bit-rate of 0.24 bpp. For presentation purposes the center 256×256 section of this image is shown in Fig. 2 having a $PSNR$ of 29.58 dB.

Reconstruction of the image using the proposed algorithm is shown in Fig. 2. The corresponding $PSNR$ is equal to 30.47dB. For comparison purposes, we show results with previous proposed algorithms in table 1. The proposed approach outperformed the non POCS approach

[9] and JPEG AC prediction recommendation [1] both in PSNR and visual quality of the image and results are better than the spatially-adaptive POCS approach [10].

6 CONCLUSIONS

A new spatially-adaptive image recovery algorithm based on the Bayesian hierarchical approach has been proposed to decode BDCT based compressed image. Using this approach it is possible to estimate the unknown hyperparameters using well grounded estimation procedures that allow to incorporate from vague to precise knowledge about the unknown parameters into the recovery process. This method is very fast. The performed tests show very good improvement in term of the $PSNR$ metric and the visual quality of the image.

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