RESTORATION OF SEVERELY BLURRED HIGH RANGE IMAGES USING COMPOUND MODELS

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ABSTRACT

In this paper we examine the use of compound Gauss Markov random fields (CGMRF) to restore severely blurred high range images. For this deblurring problem, the convergence of the Simulated Annealing (SA) and Iterative Conditional Mode (ICM) algorithms has not been established. We propose two new iterative restoration algorithms which extend the classical SA and ICM approaches. Their convergence is established and they are tested on real and synthetic images.

1. INTRODUCTION

The CGMRF theory provides a foundation for the characterization of spatial, contextual constrains on the image model using a hidden random field. The use of CGMRF was first presented in [2] using an Ising model to represent the upper level and a line process to model the abrupt transitions. Extensions to continuous range models using CGMRFs were presented in [3, 1].

In this paper we present the application of CGMRFs to the restoration of severely blurred high range images, a problem for which convergence of the SA algorithm has not been established. In section 2 we introduce the notation we use and the proposed image and noise models. Stochastic and deterministic relaxation approaches to obtain the maximum *a posteriori* (MAP) estimate are presented in section 3. In section 4 we examine the problems introduced by the blurring when SA and ICM are applied and propose two new methods to estimate the real underlying image. Examples are shown in section 5 and section 6 concludes the paper.

2. NOTATION AND MODELS

We will distinguish between f, the 'true' image which would be observed under ideal conditions and g, the observed image. The aim is to reconstruct f from g. For simplicity, we will denote by $f(i)$ the intensity of the true image at the pixel location i on the lattice. We regard f as a $p \times 1$ column vector, with entries $f(i)$. This convention applies equally to the observed image g.

Let us now introduce the CGMRF model from a simpler model, the Conditional Auto-Regression (CAR). The idea is to build a prior model consisting of two processes, one accounting for the intensity values and the other for the location of edges in the image.

Let us first describe the prior model without any edges. Our prior knowledge about the smoothness of the object luminosity distribution makes it possible to model the distribution of f by a CAR (see [5]). Thus,

$$p(f) \propto \exp\left\{-\frac{1}{2\sigma_w^2} f^T (I - \phi N) f\right\}, \qquad (1)$$

where $N_{ij} = 1$ if cells i and j are spatial neighbors (pixels at distance one), zero otherwise and ϕ just less than 0.25. The term $f^T(I-\phi N)f$ represents in matrix-vector notation the sum of squares of the values $f(i)$ minus ϕ times the sum of $f(i)f(j)$ for neighboring pixels i and j. The parameters can be interpreted by the following expressions describing the conditional distribution

$$\begin{aligned} \mathrm{E}(f(i) \mid f(j),\ j \neq i) &= \phi \sum_{j\ nhbr\ i} f(j), \\ \mathrm{var}(f(i) \mid f(j),\ j \neq i) &= \sigma_w^2, \end{aligned}$$

where '$j\ nhbr\ i$' denotes the four neighbor pixels at distance one from pixel i (see Figure 1). The parameter σ_w^2 measures the smoothness of the 'true' image.

From Eq. (1) we have

$$\begin{aligned} -\log p(f) = Z + \frac{1}{2\sigma_w^2} \sum_i f(i)\,[f(i) - \phi(Nf)(i)] \\ = \quad Z + \sum_i \phi(f(i) - f(i:+1))^2/(2\sigma_w^2) \\ + \quad \sum_i \phi(f(i) - f(i:+2))^2/(2\sigma_w^2) \\ + \quad \sum_i (1-4\phi) f^2(i)/(2\sigma_w^2), \end{aligned}$$

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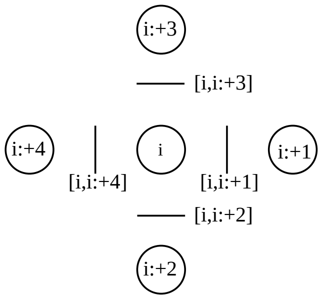

Figure 1: Image and line sites.

where $i:+1$, $i:+2$, $i:+3$, $i:+4$ denote the four image pixels around pixel i as described in figure 1, Z is a normalization constant and we assume a 'toroidal edge correction'.

This expression can be rewritten as

$$
\begin{aligned}
-\log p(f,l) &= Z \\
&+ \frac{1}{2\sigma_w^2}\sum_i \phi(f(i)-f(i+1))^2(1-l([i,(i:+1)])) \\
&+ \frac{1}{2\sigma_w^2}\sum_i \phi(f(i)-f(i+2))^2(1-l([i,(i:+2)])) \\
&+ \frac{1}{2\sigma_w^2}\sum_i [\beta l([i,(i:+1)]) + \beta l([i,(i:+2)])] \\
&+ \frac{1}{2\sigma_w^2}\sum_i (1-4\phi) f^2(i) \qquad (2)
\end{aligned}
$$

where $l([i,j]) \equiv 0$ for all i and j, and β is a scalar weight.

We now introduce a line process by simply redefining the function $l([i,j])$ as taking the value zero if neighbor pixels i and j are not separated by an active line and one otherwise. We then penalize the introduction of the line element $[i,j]$ (see figure 1) by the term $\beta l([i,j])$ since otherwise the expression in (2) would obtain its minimum value by setting all line elements equal to one. The intuitive interpretation of this line process is simple; it acts as an inhibitor or activator of the relation between two neighbor pixels depending on whether or not the pixels are separated by an edge.

In this paper we shall use this simple image model. The theory can be easily extended to more complex image model including a larger neighborhood, interactions in the line process or β being direction dependent.

A simplified but realistic noise model for many applications is the Gaussian model with zero mean and variance σ_n^2, $\mathcal{N}(0,\sigma_n^2)$. This means that the observed image corresponds to the model $g(i) = (Df)(i) + n(i) = \sum_j d(i-j)f(j) + n(i)$, where D is the $p \times p$ matrix defining the systematic blur, assumed to be known and approximated by a block circulant matrix, $n(i)$ is the additive Gaussian noise with zero mean and variance σ_n^2 and $d(j)$ are the coefficients defining the blurring function.

Then, the probability of the observed image g if f were the 'true' image is

$$
p(g \mid f) \propto \exp\left[-\frac{1}{2\sigma_n^2}\|g - Df\|^2\right]. \qquad (3)
$$

Let us now proceed to find $\hat{f}$, $\hat{l}$, the MAP estimates of f and l, that is

$$
\hat{f}, \hat{l} = \arg\max_{f,l} p(f, l \mid g). \qquad (4)
$$

3. STOCHASTIC AND DETERMINISTIC RELAXATION FOR MAP ESTIMATION

Since $p(f,l \mid g)$ is nonlinear it is extremely difficult to find $\hat{f}$ and $\hat{l}$ by any conventional method. Simulated annealing is a relaxation technique to search for MAP estimates from degraded observations (see [3] for details).

It uses the distribution

$$
\begin{aligned}
p_T(f,l \mid g) &= \frac{1}{Z_T}\exp[-\frac{1}{T}U(g \mid f) \\
&-\frac{1}{2\sigma_w^2 T}\sum_i [\phi(f(i)-f(i:+1))^2(1-l([i,(i:+1)])) \\
&+\phi(f(i)-f(i:+2))^2(1-l([i,(i:+2)])) \\
&+\beta l([i,(i:+1)]) + \beta l([i,(i:+2)]) + (1-4\phi)f^2(i)]],
\end{aligned}
$$

where T is the temperature, Z_T is a normalization constant and

$$
U(g \mid f) = \frac{1}{2\sigma_n^2}\|g - Df\|^2 .
$$

It then needs to simulate the conditional *a posteriori* density function for $l([i,j])$, given the rest of l, f and g and the conditional *a posteriori* density function for $f(i)$ given the rest of f, l and g, and also to decrease, after each sweep of the full image, the temperature T according to an annealing scheme (see [3]).

Let us now examine these conditional distributions. For this purpose we will use

$$
\begin{aligned}
F(i) &= \{f(j) : \forall j \neq i\} \\
L(u) &= \{l(v) : \forall v \neq u\}
\end{aligned}
$$

where u and v denote sites in the line process. To simulate the line process conditional *a posteriori* density function, $p_T(l([i,j]) \mid L([i,j]), f, g)$, we have

$$
\begin{aligned}
&p_T\big(l([i,j]) = 0 \mid L([i,j]), f, g\big) \propto \\
&\qquad \exp\left[-\frac{1}{T}\frac{\phi}{2\sigma_w^2}(f(i)-f(j))^2\right], \qquad (5)
\end{aligned}
$$

and

$$
p_T\big(l([i,j]) = 1 \mid L([i,j]), f, g\big) \propto \exp\left[-\frac{1}{T}\frac{\beta}{2\sigma_w^2}\right]. \qquad (6)
$$

Furthermore, for our Gaussian noise model,

$$
p_T(f(i) \mid F(i), l, g) \sim \mathcal{N}(\mu^{\underline{l}[i]}(i), T\sigma^{2\,\underline{l}[i]}(i)), \qquad (7)
$$

where

$$
\begin{aligned}
\mu^{\underline{l}[i]}(i) &= \lambda^{\underline{l}[i]}(i) \sum_{j\ nhbr\ i} \phi \frac{f(j)(1-l([i,j]))}{nn^{\underline{l}[i]}(i)} \\
&+ (1-\lambda^{\underline{l}[i]}(i))\left(\frac{(D^T g)(i) - (D^T D f)(i)}{c} + f(i)\right), \qquad (8)
\end{aligned}
$$

$$\sigma^{2\underline{l}[i]}(i) = \frac{\sigma_w^2 \sigma_n^2}{nn^{\underline{l}[i]}(i)\sigma_n^2 + c\,\sigma_w^2}, \tag{9}$$

with $c = \sum_j d(j)^2$,

$$\lambda^{\underline{l}[i]}(i) = \frac{nn^{\underline{l}[i]}(i)\ \sigma_n^2}{nn^{\underline{l}[i]}(i)\sigma_n^2 + c\,\sigma_w^2}, \tag{10}$$

$nn^{\underline{l}[i]}(i) = \sum_{j\ nhbr\ i} \phi(1 - l([i,j])) + (1 - 4\phi)$ and $\underline{l}[i]$ is the four dimensional vector representing the line process configuration around image pixel (i).

Instead of using a stochastic approach, a deterministic method can be used to search for a local maximum. Such a method instead of simulating the distributions mentioned above, it chooses the mode. An advantage of the deterministic method (ICM) is that its convergence is much faster than that of the stochastic approach. The disadvantage is the local nature of the solution obtained. This method can be seen as a particular case of simulated annealing where the temperature is always set to zero.

4. PROPOSED RESTORATION ALGORITHMS

Unfortunately, due to the presence of blurring the convergence of SA has not been established for this problem. The main problem of the methods is that, if c is small, as is the case for severely blurred images, the term $[(D^T g)(i) - (D^T Df)(i)]/c$ in Eq. (8) is highly unstable. For the ICM method the problem gets worse because sudden changes in the first stages, due to the line process, become permanent.

Let us examine intuitively and formally why we may have convergence problems with the SA and ICM procedures when severe blurring is present. Let us assume for simplicity that there is no line process and examine the iterative procedure where we update the whole image at the same time. It is important to note that this is not the parallel version of SA but an iterative procedure. We have,

$$f_t = Af_{t-1} + const, \tag{11}$$

where t is the iteration number, understood as sweep of the whole image, $\lambda = \sigma_n^2/(\sigma_n^2 + c\,\sigma_w^2)$,

$$A = \left[I - \lambda(I - \phi N) - (1 - \lambda)\frac{D^T D}{c} \right], \tag{12}$$

and $const = (1 - \lambda)[D^T D/c]g$.

For the method to converge A must be a contraction mapping. However this may not be the case. For instance, if the image suffers from severe blurring then c is close to zero and the matrix $[D^T D/c]$ has eigenvalues greater than one. Furthermore, if the image has a high dynamic range, like astronomical images where ranges $[0, 7000]$ are common, it is natural to assume that σ_w^2 is big and thus, $(1-\lambda)[D^T D/c]$ has eigenvalues greater than one. Therefore, this iterative method may not converge. It is important to note that, when there is no blurring, $c = 1$ and A is a contraction mapping.

Let us modify A in order to have a contraction. Adding $[(1 - \lambda)(1 - c)/c]f$ to both sides of Eq. (11) we have

$$\begin{aligned}(1 + [(1 - \lambda)(1 - c)/c])f_t \quad = \quad & [(1 - \lambda)(1 - c)/c]f_t \\ & + Af_{t-1} + const\end{aligned}$$

or

$$f_t = \omega f_{t-1} + (1 - \omega)[Af_{t-1} + const],$$

with $\omega = (1 - c)\sigma_w^2/(\sigma_n^2 + \sigma_w^2)$. We then have for this new iterative procedure

$$f_t = \tilde{A} f_{t-1} + (1 - \omega)const,$$

where

$$\tilde{A} = \left[I - \rho(I - \phi N) - (1 - \rho)D^T D \right],$$

with $\rho = \sigma_n^2/(\sigma_n^2 + \sigma_w^2)$, is now a contraction mapping.

Let us now examine how to modify the SA procedure. Denote by t_k the number of iteration for k sweeps of the whole image, where one iteration is understood as modifying just one pixel. We use the value of $f(i)$ obtained in the previous iteration, $f_{t_{k-1}}(i)$, and, instead of simulating from the normal distribution defined in (8), (9) to obtain the new value of $f(i)$, we simulate from the normal distribution with mean

$$\eta^{\underline{l}[i]}(i) = \omega^{\underline{l}_{t_k}[i]}(i) f_{t_{k-1}}(i) + (1 - \omega^{\underline{l}_{t_k}[i]}(i))\mu^{\underline{l}_{t_k}[i]}(i) \tag{13}$$

where $\omega^{\underline{l}_{t_k}[i]}(i) = (1 - c)\sigma_w^2/(nn^{\underline{l}_{t_k}[i]}(i)\sigma_n^2 + \sigma_w^2)$, and variance

$$\sigma^{2\,\underline{l}_{t_k}[i]}_{m}(i) = (1 - (\omega^{\underline{l}_{t_k}[i]})^2)\sigma^{2\underline{l}_{t_k}[i]}(i). \tag{14}$$

This guarantees that lemma 1 of [3] is satisfied (see [4] for details). Intuitively, the idea to use this variance, stems from the fact that we have to reduce the variance since we are not simulating from the mean value.

5. EXPERIMENTAL RESULTS

Let us examine how the modified ICM algorithm works on a synthetic star image, blurred with an atmospherical point spread function (PSF), D, given by

$$d(i) \propto \left(1 + (u^2 + v^2)/R^2\right)^{-\delta} \tag{15}$$

with $\delta = 3$, $R = 3.5$, $i = (u, v)$, and Gaussian noise with $\sigma_n^2 = 64$. If we use $\sigma_w^2 = 24415$, which is realistic for this image, and taking into account that, for the PSF defined in (15), $c = 0.02$, A defined in (12) is not a contraction. Figure 2a depicts the corrupted image. Restorations from the original and modified ICM methods with $\beta = 2$ for 100 iterations are depicted on figures 2(b, c), respectively.

The modified SA algorithm was tried on the cameraman image. The original image was blurred with an atmospherical PSF with $\delta = 3$, $R = 3$ and Gaussian noise with $\sigma_n^2 = 49$ (see figure 3a). The restored image and the line process for 2000 iterations, with $\sigma_w^2 = 400$ and $\beta = 3$, are depicted on figures 3(b, c), respectively. We also ran the SA algorithm on this image. It did not converge even without the line process.

6. CONCLUSIONS

In this paper we have presented two new methods that can be used to restore high dynamic range images in the presence of severe blurring. These methods extend the classical ICM and SA procedures, so that convergence of the algorithms is now guaranteed. The experimental results verify the derived theoretical results. Further extensions of the algorithms are under consideration.

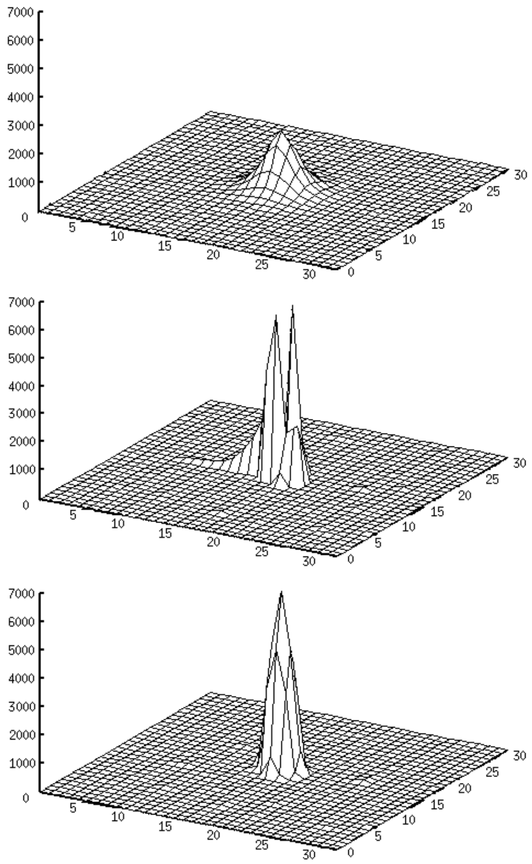

Figure 2: a) Observed image. b) ICM restoration. c) Restoration with the proposed ICM method.

Figure 3: a) Observed image. b) Modified Simulated Annealing. c) Obtained line process.

7. REFERENCES

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