Parameter Estimation in Regularized Reconstruction of BDCT Compressed Images for Reducing Blocking Artifacts

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ABSTRACT

High compression ratios for both still images and sequences of images are usually achieved by discarding information represented by block discrete cosine transform (BDCT) coefficients which is considered unimportant. This compression procedure yields images that exhibit annoying blocking artifacts. In this paper we examine the reconstruction of BDCT compressed images which results in the removal of the blocking artifact.

The method we propose for the reconstruction of such images, is based on a hierarchical Bayesian approach. With such an approach image and degradation models are required. In addition, unknown hyperparameters, usually the noise and image variances, have to be estimated in advanced or simultaneously with the reconstructed image. We show how to introduce knowledge about these parameters into the reconstruction procedure. The proposed algorithm is tested expreimentally.

1 INTRODUCTION

Block-transform coding is by far the most popular approach for image compression. Evidence of this fact is that both the Joint Photograph Experts Group (JPEG) and the Motion Pictures Experts Group (MPEG) recommend the use of the block discrete cosine transform (BDCT) for compressing still and sequences of motion images, respectively[11,20] . However, some researchers on image compression using wavelets believe that JPEG does not represent the "state of art" standard. For instance, Hilton, et al.[10] claim that "The reconstruction quality of wavelet compressed images has already moved well beyond the capabilities of JPEG, which is the current international standard for image compression". Further evidence for this is supplied by the FBI wavelet image compression standard, which provides good quality images for compression rates around 20:1[2]. Such claims are usually based on the existence of the blocking artifact in JPEG or any block-transform based compressed image at high compression ratios.

This artifact manifests itself as an artificial discontinuity between adjacent blocks. It is a direct result of the independent processing of the blocks which does not take into account the between-block pixel correlations. It constitutes a serious bottleneck for many important visual communication applications that require visually pleasing images at very high compression ratios. The recent progress in VLSI technology makes us believe that the incorporation of recovery algorithms at the decoders is a very promising approach to bridging the conflicting requirements of high-quality images and high compression ratios.

In the past various algorithms have been proposed to improve quality of block-transform compressed images in the decoder without increasing the bit-rate. In the JPEG standard[11,20] a technique for predicting the AC coefficients is recommended in Annex-K.8.2, as an option at the decoder in order to suppress the between block discontinuities of the decoded images. However, in areas with sharp intensity transitions such a prediction scheme fails. Stevenson[22] assumes a probabilistic model and the compressed image is reconstructed using a maximum *a posteriori* (MAP) approach. However the prior distribution used for the original image is not spatially-varying. Yang, *et al.*[23] proposed a set theoretic approach based on the theory of POCS and a constrained least-squares approach based on regularization both using spatially-invariant prior knowledge to reconstruct the compressed image along with the transmitted data. Another MAP approach based on a compound Gauss Markov image model and mean field annealing was proposed by Özcelik *et al.*[19]. In another paper, Yang, *et al.*[24] proposed a POCS based recovery algorithm that uses a spatially-adaptive constraint that enforces between-block smoothness.

The method proposed by Yang *et al.*[24] requires the estimation of the regularization parameter. In this paper we formulate their algorithm within the Bayesian paradigm and perform the estimation of the regularization parameters, representing the image and noise variances, following the hierarchical Bayesian approach[16,17]. We show how, following this approach, it is possible to incorporate knowledge about the unknown hyperparameters ranging from vague to very precise, into the algoritm. The resulting methods are very suitable for VLSI implementation.

The rest of the paper is organized as follows. In section 2 the mathematical background for this paper is established. Section 3 describes the prior and noise models for the reconstruction problem. In section 4 we describe the use of the hierarchical approach to the hyperparameter estimation and the recovery algorithm is presented. Experimental results are presented in section 5 and, finally, in section 6 we present our conclusions.

2 MATHEMATICAL DEFINITION OF THE RECOVERY PROBLEM

Throughout this paper a digital $N \times N$ image is treated as a $N^2 \times 1$ vector in the R^{N^2} space by lexicographic ordering either the rows or the columns. The l_2 norm is used as the distance measure. The BDCT is viewed as a linear transformation from R^{N^2} to R^{N^2}. Then, for an image $\mathbf{f}$ we can write

$$\mathbf{F} = \mathbf{B}\mathbf{f}, \tag{1}$$

where $\mathbf{F}$ is the BDCT of $\mathbf{f}$ and $\mathbf{B}$ is the BDCT matrix. To achieve a bit-rate reduction for transmission, each element of $\mathbf{F}$ is quantized. This quantization operator can be described mathematically by a mapping or an operator from R^{N^2} to R^{N^2}. Let $\mathcal{Q}$ denote this operator; the input-output relation of the coder can be modeled by

$$\mathbf{G} = \mathcal{Q}\mathbf{B}\mathbf{f}. \tag{2}$$

Due to the unitary property of the DCT matrices, the BDCT matrix is also unitary and the inverse transform can be simply expressed by $\mathbf{B}^t$ where t denotes the transpose of a matrix. In the receiver only the quantized BDCT coefficients $\mathbf{G}$ are available and the output of a conventional decoder is

$$\mathbf{g} = \mathbf{B}^t\mathbf{G}. \tag{3}$$

The reconstruction problem calls for finding an estimate of $\mathbf{f}$ given $\mathbf{g}$, $\mathcal{Q}$ and, possibly, knowledge about $\mathbf{f}$. In this work we propose to reconstruct the artificial block boundaries, where the blocking artifact is more visible, keeping the rest of the image unchanged. Following the Bayesian paradigm a degradation model is needed which describes how the blocky image is obtained from the real one. In addition, a model describing our knowledge about the original image, the image before coding is also required. Let us study these models in detail.

3 IMAGE AND NOISE MODELS

We can represent an $N \times N$ image $\mathbf{f}$ as

$$\mathbf{f} = \{\mathbf{f}_c(1)\ \mathbf{f}_c(2)\ \ldots\ \mathbf{f}_c(N)\}, \tag{4}$$

where $\mathbf{f}_c(i)$ represent the ith column of the image. The same convention can be applied to $\mathbf{g}$. If we assume that 8×8 blocks are used and $N = 512$, we can define the columns at the block boundaries as

$$\mathbf{u}_c = \begin{bmatrix} \mathbf{f}_c(8) \\ \mathbf{f}_c(16) \\ \vdots \\ \mathbf{f}_c(504) \end{bmatrix}, \quad \mathbf{v}_c = \begin{bmatrix} \mathbf{f}_c(9) \\ \mathbf{f}_c(17) \\ \vdots \\ \mathbf{f}_c(505) \end{bmatrix}, \quad \mathbf{x}_c = \begin{bmatrix} \mathbf{g}_c(8) \\ \mathbf{g}_c(16) \\ \vdots \\ \mathbf{g}_c(504) \end{bmatrix}, \quad \mathbf{y}_c = \begin{bmatrix} \mathbf{g}_c(9) \\ \mathbf{g}_c(17) \\ \vdots \\ \mathbf{g}_c(505) \end{bmatrix}. \tag{5}$$

In a similar way we can represent $\mathbf{f}$ as

$$\mathbf{f} = \{\mathbf{f}_r(1)\ \mathbf{f}_r(2)\ \ldots\ \mathbf{f}_r(N)\}, \tag{6}$$

where $\mathbf{f}_r(i)$ represent the ith row of the image, in column form, and define the rows at the block boundaries as column vectors form as

$$\mathbf{u}_r = \begin{bmatrix} \mathbf{f}_r(8) \\ \mathbf{f}_r(16) \\ \vdots \\ \mathbf{f}_r(504) \end{bmatrix}, \quad \mathbf{v}_r = \begin{bmatrix} \mathbf{f}_r(9) \\ \mathbf{f}_r(17) \\ \vdots \\ \mathbf{f}_r(505) \end{bmatrix}, \quad \mathbf{x}_r = \begin{bmatrix} \mathbf{g}_r(8) \\ \mathbf{g}_r(16) \\ \vdots \\ \mathbf{g}_r(504) \end{bmatrix}, \quad \mathbf{y}_r = \begin{bmatrix} \mathbf{g}_r(9) \\ \mathbf{g}_r(17) \\ \vdots \\ \mathbf{g}_r(505) \end{bmatrix}. \tag{7}$$

In order to capture the vertical local properties of the image we define a $(512 * 63) \times (512 * 63)$ diagonal matrix, $\mathbf{W}_c$, of the form[24]

$$\mathbf{W}_c = \begin{bmatrix} \omega_c(1) & 0 & 0 & \cdots & 0 \\ 0 & \omega_c(2) & 0 & \cdots & \vdots \\ 0 & 0 & \ddots & 0 & \vdots \\ \vdots & \vdots & 0 & \ddots & 0 \\ 0 & \cdots & \cdots & 0 & \omega_c(512*63) \end{bmatrix}, \tag{8}$$

where the $\omega_c(i)$'s, $i = 1, 2, \ldots, (512 * 63)$ will weight each block boundary pixel. Analogously, we can define $\mathbf{W}_r$ to capture the horizontal local properties of the image.

In order to obtain the weights $\omega_c(i)$ and $\omega_r(i)$ the pixel at location (i) is treated as a random variable with mean $\mu(i)$ and variance $\sigma^2(i)$. The mean serves as a measure of the local brightness and the variance as a measure of the local detail at the pixel location (i). Several forms of weighting functions has been previously suggested[24]. An example of such a function is

$$\omega_c(i) = \log\left(1 + \frac{\sqrt{\mu(i)}}{1+\sigma(i)}\right), \tag{9}$$

a compressed range function which decrease as a function of $\sigma(i)$ and captures the intensity changes, since the blocking artifacts seems to be more visible in bright rather than in dark areas of the image (see section 5 for details).

With the above definitions, the prior knowledge about the smoothness in the block boundaries of the image has the form

$$p(\mathbf{f}|\alpha_c, \alpha_r) \propto \alpha_c^p \alpha_r^p \exp\left\{-\alpha_c \parallel \mathbf{W}_c(\mathbf{u}_c - \mathbf{v}_c) \parallel^2 - \alpha_r \parallel \mathbf{W}_r(\mathbf{u}_r - \mathbf{v}_r) \parallel^2\right\}, \tag{10}$$

with $p = 512 \times 63$ and α_c and α_r measures of the roughness between two block boundary columns and rows.

Fidelity to the data at these block boundaries is expressed by

$$p(\mathbf{g}|\mathbf{f}, \beta_c, \beta_r) \propto \beta_c^p \beta_r^p \exp\left\{-\frac{1}{2}\beta_c \parallel \mathbf{u}_c - \mathbf{x}_c \parallel^2 - \frac{1}{2}\beta_c \parallel \mathbf{v}_c - \mathbf{y}_c \parallel^2 - \frac{1}{2}\beta_r \parallel \mathbf{u}_r - \mathbf{x}_r \parallel^2 - \frac{1}{2}\beta_r \parallel \mathbf{v}_r - \mathbf{y}_r \parallel^2\right\}, \tag{11}$$

where β_c and β_r are defined as $\beta_c^{-1} = \sigma^2_{noise\,c}$ (the noise variance in the vertical direction) and $\beta_r^{-1} = \sigma^2_{noise\,r}$ (the noise variance in the horizontal direction), respectively.

If α_c, β_c, α_r and β_r are known, then following the Bayesian paradigm it is customary to select, as the reconstruction of $\mathbf{f}$, the image $\mathbf{f}_{(\alpha_c,\beta_c,\alpha_r,\beta_r)}$ defined by

$$\begin{aligned}\mathbf{f}_{(\alpha_c,\beta_c,\alpha_r,\beta_r)} = \arg\Big\{\min_{\mathbf{f}}\Big\{&\alpha_c \parallel \mathbf{W}_c(\mathbf{u}_c - \mathbf{v}_c) \parallel^2 + \alpha_r \parallel \mathbf{W}_r(\mathbf{u}_r - \mathbf{v}_r) \parallel^2 + \frac{1}{2}\beta_c \parallel \mathbf{u}_c - \mathbf{x}_c \parallel^2 + \\ &\frac{1}{2}\beta_c \parallel \mathbf{v}_c - \mathbf{y}_c \parallel^2 + \frac{1}{2}\beta_r \parallel \mathbf{u}_r - \mathbf{x}_r \parallel^2 + \frac{1}{2}\beta_r \parallel \mathbf{v}_r - \mathbf{y}_r \parallel^2\Big\}\Big\}. \end{aligned} \tag{12}$$

Two important points are worth noting. We are using different hyperparameters for the rows and columns,since the local characteristics may not be the same for rows and columns. It is important to note that $\mathbf{u}_c$ and $\mathbf{v}_c$ have pixels in common with $\mathbf{u}_r$ and $\mathbf{v}_r$ (where rows and columns intersect).

4 PROPOSED ALGORITHM

It is obvious that in order to estimate the original image, that is, in order to solve eq. (12) we need to know or estimate the unknown hyperparameters. Yang *et al.*[23,24] proposed empirical procedures to estimate these parameters within the POCS and CLS approaches to the reconstruction problem. In this paper we follow the so called hierarchical Bayesian paradigm. A paradigm where prior knowledge about the unknown hyperparameters can be incorporated into the algorithm if available or they can be estimated without any prior knowledge.

The hierarchical Bayesian paradigm is being currently applied to many areas of research related to image analysis. Buntine[3] applies this theory to the construction of classification trees. Spiegelhalter and Lauritzen[21] have applied the hierarchical Bayesian framework to the problem of refining probabilistic networks. Buntine[4] and Cooper and Herkovsits[6] have used the same framework to address the problem of constructing such networks. MacKay[12] and Buntine and Weigund[5] use the full Bayesian framework in backpropagation networks and MacKay[13], following Gull[7], applies this framework to interpolation problems.

This hierarchical paradigm applied to our problem uses the global probability defined as

$$p(\alpha_c, \beta_c, \alpha_r, \beta_r, \mathbf{f}, \mathbf{g}) = p(\alpha_c)p(\beta_c)p(\alpha_r)p(\beta_r)p(\mathbf{f}|\alpha_c, \alpha_r)p(\mathbf{g}|\mathbf{f}, \beta_c, \beta_r). \tag{13}$$

Let us examine all the term involved in this equation. Having defined in the previous section the image and degradation models, that is, $p(\mathbf{f}|\alpha_c, \alpha_r)$ and $p(\mathbf{g}|\mathbf{f}, \beta_c, \beta_r)$, we have to specify the probabilities on the hyperparameters.

In situations without prior information what is needed is a non informative prior on the hyperparameters, $\alpha_c, \beta_c, \alpha_r$ and β_r (the term non informative is meant to imply that no information about the hyperparameters is contained in the priors). For the problem at hand we can use improper non informative priors $p(\omega) \propto const$ over $[0, \infty)$ where ω denotes a hyperparameter.

However, it is also possible, to incorporate precise prior knowledge about the value of the noise and prior variances. To do so we use as hyperprior the gamma distribution defined by

$$p(\omega) \propto \omega^{l/2-1} \exp[-a(l-2)\omega],$$

where a is a constant whose meaning will be made precise later, and l is a non negative quantity. This distribution has the following properties

$$E[w] = \frac{l}{2a(l-2)} \qquad \text{and} \qquad Var[w] = \frac{l}{2a^2(l-2)^2}.$$

So, the mean of w, the inverse of the prior or noise variance is, for l large, approximately equal to $1/2a$, and its variance decreases when l is increased; l can then be understood as a measure of the certainty on the knowledge about the prior or noise variances[16,18].

After having defined $p(\alpha_c, \beta_c, \alpha_r, \beta_r, \mathbf{f}, \mathbf{g})$, the Bayesian analysis is performed. The evidence framework integrates over the parameters $\mathbf{f}$ to give the evidence $p(\alpha_c, \beta_c, \alpha_r, \beta_r|\mathbf{g})$ and then this likelihood is maximized over the hyperparameters.

This approach works as follows. First, select $\hat{\alpha}_c, \hat{\beta}_c, \hat{\alpha}_r$ and $\hat{\beta}_r$ as

$$\hat{\alpha}_c, \hat{\beta}_c, \hat{\alpha}_r, \hat{\beta}_r = \arg \max_{\alpha_c, \beta_c, \alpha_r, \beta_r} p(\alpha_c, \beta_c, \alpha_r, \beta_r|\mathbf{g}),$$

and then select $\mathbf{f}_{(\hat{\alpha}_c, \hat{\beta}_c, \hat{\alpha}_r, \hat{\beta}_r)}$, defined in eq. (12), as the reconstructed image.

For the sake of simplicity, in order to estimate $\hat{\alpha}_c$, $\hat{\beta}_c$,$\hat{\alpha}_r$ and $\hat{\beta}_r$ we suppose that column block boundary pixels are not related to row block boundary pixels although, as we have said, $\mathbf{u}_c$ and $\mathbf{v}_c$ have pixels in common with $\mathbf{u}_r$ and $\mathbf{v}_r$. This assumption implies that we can estimate prior and noise variances for the column block boundaries independently of the row block boundary values. The same assumption is adopted with respect to the row block boundaries.

Having taken into account the previous assumptions, the hierarchical approach works by selecting $\hat{\alpha}_c$ and $\hat{\beta}_c$ as

$$\hat{\alpha}_c, \hat{\beta}_c = \arg \max_{\alpha_c, \beta_c} p(\alpha_c, \beta_c|\mathbf{g}),$$

and, $\hat{\alpha}_r$ and $\hat{\beta}_r$ as

$$\hat{\alpha}_r, \hat{\beta}_r = \arg \max_{\alpha_r, \beta_r} p(\alpha_r, \beta_r|\mathbf{g}),$$

then selecting $\mathbf{f}_{(\hat{\alpha}_c, \hat{\beta}_c)}$ as

$$\mathbf{f}_{(\hat{\alpha}_c, \hat{\beta}_c)} = \arg \left\{ \min_{\mathbf{u}_c, \mathbf{v}_c} \left\{ \hat{\alpha}_c \parallel \mathbf{W}_c(\mathbf{u}_c - \mathbf{v}_c) \parallel^2 + \frac{1}{2}\hat{\beta}_c \parallel \mathbf{u}_c - \mathbf{x}_c \parallel^2 + \frac{1}{2}\hat{\beta}_c \parallel \mathbf{v}_c - \mathbf{y}_c \parallel^2 \right\} \right\}. \tag{14}$$

and $\mathbf{f}_{(\hat{\alpha}_r, \hat{\beta}_r)}$ as

$$\mathbf{f}_{(\hat{\alpha}_r, \hat{\beta}_r)} = \arg \left\{ \min_{\mathbf{u}_r, \mathbf{v}_r} \left\{ \hat{\alpha}_r \parallel \mathbf{W}_r(\mathbf{u}_r - \mathbf{v}_r) \parallel^2 + \frac{1}{2}\hat{\beta}_r \parallel \mathbf{u}_r - \mathbf{x}_r \parallel^2 + \frac{1}{2}\hat{\beta}_r \parallel \mathbf{v}_r - \mathbf{y}_r \parallel^2 \right\} \right\}. \tag{15}$$

The final reconstruction consists of $\mathbf{u}_c(\hat{\alpha}_c, \hat{\beta}_c)$, $\mathbf{v}_c(\hat{\alpha}_c, \hat{\beta}_c)$, $\mathbf{u}_r(\hat{\alpha}_c, \hat{\beta}_c)$ and $\mathbf{v}_r(\hat{\alpha}_c, \hat{\beta}_c)$ and, where the rows and columns cross, the reconstruction is obtained as the mean of the values obtained from the row and column reconstruction. It is important to note that the method can also be implemented taking into account the correlation between rows and columns and, these correlations amount to deal with 4×4 matrices that can be easily inverted[14].

In the hierarchical model we have to define the hyperprior. In this paper we shall use improper priors, although more complex models can be used. See Ref.[17] for a general study and Ref.[14] for its application to image reconstruction.

With the improper hyperpriors, the method proposed amounts to selecting $\hat{\alpha}_c, \hat{\beta}_c$ as the maximum likelihood estimates, *mle*, of α_c, β_c from $p(\mathbf{g}|\alpha_c, \beta_c)$. Let us proceed to examine the estimation process in detail. Let us fix α_c and β_c and expand $M(\mathbf{f}, \mathbf{g}|\alpha_c, \beta_c) = \alpha_c \parallel \mathbf{W}_c(\mathbf{u}_c - \mathbf{v}_c) \parallel^2 + \frac{1}{2}\beta_c \parallel \mathbf{u}_c - \mathbf{x}_c \parallel^2 + \frac{1}{2}\beta_c \parallel \mathbf{v}_c - \mathbf{y}_c \parallel^2$ around $\mathbf{f}_{(\alpha_c,\beta_c)}$. We then have[7,13],

$$
\begin{aligned}
p(\alpha_c, \beta_c|\mathbf{g}) &\propto p(\mathbf{g}|\alpha_c, \beta_c) && (16)\\
&\propto \frac{\exp\{-M(\mathbf{f}_{(\alpha_c,\beta_c)}, \mathbf{g}|\alpha_c, \beta_c)\}}{\alpha_c^p \beta_c^p} \int_{\mathbf{f}} \exp\left\{-\frac{1}{2}(\mathbf{f} - \mathbf{f}_{(\alpha_c,\beta_c)})^t \mathbf{Q}(\alpha_c, \beta_c)(\mathbf{f} - \mathbf{f}_{(\alpha_c,\beta_c)})\right\} d\mathbf{f} && (17)\\
&= \frac{\exp\{-M(\mathbf{f}_{(\alpha_c,\beta_c)}, \mathbf{g}|\alpha_c, \beta_c)\} det\, \mathbf{Q}(\alpha_c, \beta_c)^{-1/2}}{\alpha_c^p \beta_c^p}, && (18)
\end{aligned}
$$

where $\mathbf{Q}(\alpha_c, \beta_c)$ is a $(512 * 63 * 2) \times (512 * 63 * 2)$ matrix which can be represented as a $(512 * 63) \times (512 * 63)$ diagonal matrix with elements, $\mathbf{q}_{i,i}(\alpha_c, \beta_c)$, defined as

$$
\mathbf{q}_{i,i}(\alpha_c, \beta_c) = \begin{bmatrix} 2\alpha_c\omega_c^2(i) + \beta_c & -2\alpha_c\omega_c^2(i) \\ -2\alpha_c\omega_c^2(i) & 2\alpha_c\omega_c^2(i) + \beta_c \end{bmatrix}. \qquad (19)
$$

Then,

$$
det\, \mathbf{Q}(\alpha_c, \beta_c)^{-1/2} = \prod_{i=1}^{512*63} det\, [\mathbf{q}_{i,i}(\alpha_c, \beta_c)]^{-1/2} = \prod_{i=1}^{512*63} \left(4\alpha_c\omega_c^2(i)\beta_c + \beta_c^2\right)^{-1/2}. \qquad (20)
$$

Differentiating $-\log p(\alpha_c, \beta_c|\mathbf{g})$ with respect to α_c and β_c we have

$$
\begin{aligned}
\parallel \mathbf{W}_c(\mathbf{u}_{c(\alpha_c,\beta_c)} - \mathbf{v}_{c(\alpha_c,\beta_c)}) \parallel^2 + \sum_{i=1}^{512*63} \frac{2\beta_c\omega_c^2(i)}{4\alpha_c\omega_c^2(i)\beta_c + \beta_c^2} &= p/\alpha_c && (21)\\
\frac{1}{2} \parallel \mathbf{x}_c - \mathbf{u}_{c(\alpha_c,\beta_c)} \parallel^2 + \frac{1}{2} \parallel \mathbf{y}_c - \mathbf{v}_{c(\alpha_c,\beta_c)} \parallel^2 + \sum_{i=1}^{512*63} \frac{2\alpha_c\omega_c^2(i) + \beta_c}{4\alpha_c\omega_c^2(i)\beta_c + \beta_c^2} &= p/\beta_c. && (22)
\end{aligned}
$$

Following the same steps as above, we obtain for α_r and β_r

$$
\begin{aligned}
\parallel \mathbf{W}_r(\mathbf{u}_{r(\alpha_r,\beta_r)} - \mathbf{v}_{r(\alpha_r,\beta_r)}) \parallel^2 + \sum_{i=1}^{512*63} \frac{2\beta_r\omega_r^2(i)}{4\alpha_r\omega_r^2(i)\beta_r + \beta_r^2} &= p/\alpha_r && (23)\\
\frac{1}{2} \parallel \mathbf{x}_r - \mathbf{u}_{r(\alpha_r,\beta_r)} \parallel^2 + \frac{1}{2} \parallel \mathbf{y}_r - \mathbf{v}_{r(\alpha_r,\beta_r)} \parallel^2 + \sum_{i=1}^{512*63} \frac{2\alpha_r\omega_r^2(i) + \beta_r}{4\alpha_r\omega_r^2(i)\beta_r + \beta_r^2} &= p/\beta_r. && (24)
\end{aligned}
$$

Furthermore, given α_c, β_c, α_r and β_r, $\mathbf{f}_{(\alpha_c,\beta_c)}$ and $\mathbf{f}_{(\alpha_r,\beta_r)}$ are estimated by differentiating eq. (14) with respect to $\mathbf{u}_c$, $\mathbf{v}_c$ and eq. (15) with respect to $\mathbf{u}_r$ and $\mathbf{v}_r$, that is,

$$
2\alpha_c\mathbf{W}_c^t\mathbf{W}_c(\mathbf{u}_c - \mathbf{v}_c) + \beta_c(\mathbf{u}_c - \mathbf{x}_c) = 0
$$

$$\begin{aligned}
2\alpha_c \mathbf{W}_c^t \mathbf{W}_c(\mathbf{v}_c - \mathbf{u}_c) + \beta_c(\mathbf{v}_c - \mathbf{y}_c) &= 0 \\
2\alpha_r \mathbf{W}_r^t \mathbf{W}_r(\mathbf{u}_r - \mathbf{v}_r) + \beta_r(\mathbf{u}_r - \mathbf{x}_r) &= 0 \\
2\alpha_r \mathbf{W}_r^t \mathbf{W}_r(\mathbf{v}_r - \mathbf{u}_r) + \beta_r(\mathbf{v}_r - \mathbf{y}_r) &= 0.
\end{aligned} \tag{25}$$

Taking into account that

$$\begin{array}{ll}
\mathbf{u}_c + \mathbf{v}_c = \mathbf{x}_c + \mathbf{y}_c, & [4\alpha_c \mathbf{W}_c^t \mathbf{W}_c + \beta_c \mathbf{I}](\mathbf{u}_c - \mathbf{v}_c) = \beta_c(\mathbf{x}_c - \mathbf{y}_c), \\
\mathbf{u}_r + \mathbf{v}_r = \mathbf{x}_r + \mathbf{y}_r, & [4\alpha_r \mathbf{W}_r^t \mathbf{W}_r + \beta_r \mathbf{I}](\mathbf{u}_r - \mathbf{v}_r) = \beta_r(\mathbf{x}_r - \mathbf{y}_r),
\end{array} \tag{26}$$

from eq. (25), results in

$$\mathbf{u}_c(i) = \frac{1}{2}[1 + \beta_c(\beta_c + 4\alpha_c\omega_c^2(i))^{-1}]\mathbf{x}_c(i) + \frac{1}{2}[1 - \beta_c(\beta_c + 4\alpha_c\omega_c^2(i))^{-1}]\mathbf{y}_c(i) \tag{27}$$

$$\mathbf{v}_c(i) = \frac{1}{2}[1 - \beta_c(\beta_c + 4\alpha_c\omega_c^2(i))^{-1}]\mathbf{x}_c(i) + \frac{1}{2}[1 + \beta_c(\beta_c + 4\alpha_c\omega_c^2(i))^{-1}]\mathbf{y}_c(i) \tag{28}$$

$$\mathbf{u}_r(i) = \frac{1}{2}[1 + \beta_r(\beta_r + 4\alpha_c\omega_r^2(i))^{-1}]\mathbf{x}_r(i) + \frac{1}{2}[1 - \beta_r(\beta_r + 4\alpha_r\omega_r^2(i))^{-1}]\mathbf{y}_r(i) \tag{29}$$

$$\mathbf{v}_r(i) = \frac{1}{2}[1 - \beta_r(\beta_r + 4\alpha_r\omega_r^2(i))^{-1}]\mathbf{x}_r(i) + \frac{1}{2}[1 + \beta_r(\beta_r + 4\alpha_r\omega_r^2(i))^{-1}]\mathbf{y}_r(i). \tag{30}$$

In summary, the following iterative algorithm can be used to recover $\mathbf{f}$, an image with reduced blocking artifact (this method can also be considered as an EM procedure[14]).

1. Set $\mathbf{f}_0 = \mathbf{g}$.
2. Choose α_c^0, β_c^0, α_r^0 and β_r^0.
3. For $k = 1, 2, \ldots$
   (a) Estimate α_c^k, β_c^k, α_r^k and β_r^k by substituting α_c^{k-1}, β_c^{k-1}, α_r^{k-1} and β_r^{k-1} in the left hand side of equations (21), (22), (23) and (24), respectively.
   (b) For $i = 1, 2, \ldots, 512 * 63$, compute $\mathbf{u}_c^k(i)$, $\mathbf{v}_c^k(i)$, $\mathbf{u}_c^k(i)$ and $\mathbf{v}_c^k(i)$ from equations (27), (28), (29) and (30), respectively
4. Goto 3 until $\| \mathbf{f}^k - \mathbf{f}^{k-1} \|$ is less than a prescribed bound.

Before testing our method in the next section it is important to note that we can also use iterative schemes for the estimation of the hyperparameters as proposed by Kang and Katsaggelos[9] . This method can also be cast within the Bayesian paradigm[17].

5 TEST EXAMPLES

In this section, experiments are presented in order to test the proposed recovery algorithm. The 512×512 "Lena" image is used. The image was compressed using a JPEG based coder-decoder with the quantization table shown in Fig. 1, which yields a bit-rate of 0.24 bpp. Each entry in the quantization table denotes the quantizer step-size for the corresponding DCT coefficient[11,20] . For presentation purposes the center 256×256 section of this image is shown in Fig. 2.

50	60	70	70	90	120	255	255
60	60	70	96	130	255	255	255
70	70	80	120	200	255	255	255
70	96	120	145	255	255	255	255
90	130	200	255	255	255	255	255
120	255	255	255	255	255	255	255
255	255	255	255	255	255	255	255
255	255	255	255	255	255	255	255

Figure 1: Quantification table used for JPEG based compression.

Figure 2: 256×256 center section of blocky "Lena" from JPEG based compression at .24 bpp

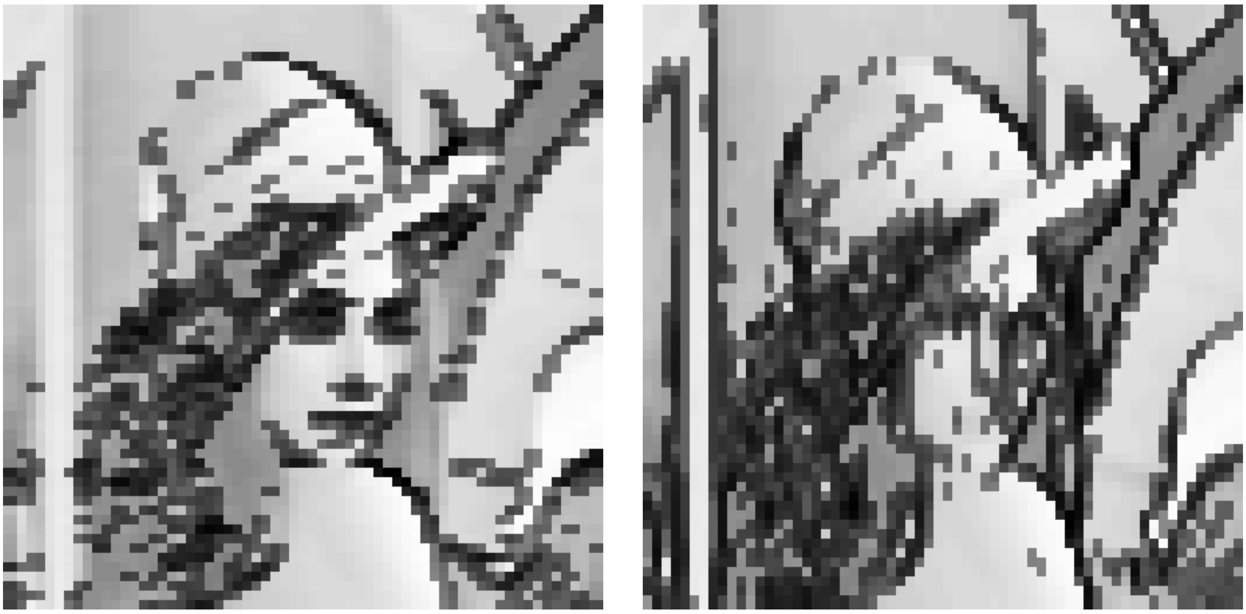
We use the peak-signal-to noise-ratio ($PSNR$) as an objective measure of distance between the reconstructed image $\hat{\mathbf{f}}$ and the original image $\mathbf{f}$. For $N \times N$ images with $[0, 255]$ gray-level range, the $PSNR$ is defined in dB by

$$PSNR = 10 \log_{10} \left[\frac{N^2 \times 255^2}{\| \hat{\mathbf{f}} - \mathbf{f} \|^2} \right]. \tag{31}$$

The $PSNR$ of the blocky image in Fig 2 is 29.58 dB.

The weights in eq. (8) are computed based on eq. (9) using the transmitted transform data. Since we have two weight matrices, one for the columns, $\mathbf{W}_c$, and another for the rows, $\mathbf{W}_r$, both means μ_c and μ_r, corresponding to $\mathbf{W}_c$ and $\mathbf{W}_r$, respectively, and both variances σ_c^2 and σ_r^2 must be estimated. The weights $\omega_c(i)$ and $\omega_r(i)$ used were constant within a 8×8 region surrounding a vertical or horizontal block boundary, respectively.

Since the DC coefficient in each block is the average (to a constant) of the pixel intensity within this block, the mean μ_c was estimated as follow: Let us consider the vertical boundary l_c and let DC_L, DC_R denote the DC coefficient of its left and right block, respectively. Then, the estimate of the mean $\mu_c(l_c)$ used for the computation



(a) (b)

Figure 3: Weights of the blocky "Lena" image: (a) vertical, (b) horizontal.

of the weights in $\mathbf{W}_c$ is given by

$$\hat{\mu}_c(l_c) = \frac{DC_L + DC_R}{2 \times 8^2}. \tag{32}$$

A similar equation is used to estimate the mean $\mu_r(l_r)$, used in the computation of the weights $\mathbf{W}_r$ taking into account, in this case, the DC coefficients of the blocks above and below this horizontal boundary.

The variance $\sigma_c^2(l_c)$ at the vertical block boundary l_c is estimated by

$$\hat{\sigma}_c^2(l_C) = \frac{VAC_L + VAC_R}{2 \times 8^2}, \tag{33}$$

where VAC_L and VAC_R are the sums of the squared AC coefficients in the first column of the blocks to the left and to the right of the boundary l_c, respectively. In a similar fashion $\sigma_r^2(l_r)$ is estimated using the sums of the squared AC coefficients in the first row of the blocks above and below this horizontal boundary. After proper scaling, the weight maps for the "Lena" image corresponding to $\mathbf{W}_c$ and $\mathbf{W}_r$ are shown in Figs. 3(a) and (b), respectively. In the bright areas blocking artifacts are more visible than in the dark areas.

Reconstruction of Fig. 2 using the proposed algorithm, with the weights shown in Fig. 3(a) and (b), is shown in Fig. 4. The corresponding $PSNR$ is equal to 30.45dB. For comparison purposes, we show results with previous proposed algorithms. Using the nonadaptive POCS approach[23] the $PSNR$ is equal to 30.32dB and using the spatially-adaptive POCS approach[24] the $PSNR$ is equal to 30.43dB . The reconstructed image using JPEG AC prediction recommendation in Annex-K.8.2[11] has a $PSNR$ of 29.52dB.

The proposed approach outperformed the non POCS approach[23] and JPEG AC prediction recommendation[11] both in PSNR and visual quality of the image and results are comparable to the spatially-adaptive POCS approach[24] . In table 1 we furnish the PSNR results using the quantization table in fig. 1.

Figure 4: Reconstructed image using the proposed algorithm.

Algorithm	$PSNR$
JPEG Blocky	29.58dB
AC Prediction	29.52dB
POCS Nonadaptive	30.32dB
POCS Adaptive	30.43dB
Proposed Algorithm	30.45dB

Table 1: $PSNR$ of the reconstructed images with different algorithms.

6 CONCLUSIONS

A new spatially-adaptive image recovery algorithm has been proposed to decode BDCT based compressed image. This algorithm is based on the Bayesian hierarchical approach to image reconstruction. Using this approach it is possible to estimate the unknown hyperparameters using well grounded estimation procedures that allow to incorporate from vague to precise knowledge about the unknown parameters into the recovery process.

This method is very fast. The performed tests show very good improvement in term of the $PSNR$ metric and the visual quality of the image. It is mentioned here that the $PSNR$ metric may not be appropiate for this application, since the removal of the blocking artifact is the primary objective. There may be therefore cases where the visual quality of the processed image improves considerably, whree as the $PSNR$ value remains the same or even decreases. Objective measures which specifically measure the removal of blocking artifacts need to be used.

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