SPECT IMAGE RECONSTRUCTION USING COMPOUND MODELS

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ABSTRACT

SPECT (Single Photon Emission Computed Tomography) is used in nuclear medicine to determine the distribution of a radioactive isotope within a patient from tomographic views or projection data. These images are severely degraded due to the presence of noise and several physical factors like attenuation and scattering. In this paper we use, within the Bayesian framework, a Compound Gauss Markov Random Field (CGMRF) as prior model to reconstruct such images. In order to find the Maximum *a Posteriori* (MAP) estimate we propose a new iterative method, which is stochastic for the line process and deterministic for the reconstruction. The proposed method is tested and compared with other reconstruction methods on both synthetic and real SPECT images.

1. INTRODUCTION

SPECT images are observation data acquired by a gamma-camera following an orbit around the patient's body, at regularly spaced angles. At each position, the registered photon counts at the gamma-camera are conveniently processed and stored as discrete two-dimensional images. A reconstructed image is the discrete representation of a slice or cross section of the isotope distribution within the patient, transversal to the gamma-camera rotation axis.

Bayesian reconstruction methods have been extensively used to reconstruct medical images since they can improve the reconstructions with respect to the classical, non statistical methods, such as FBP (Filtered Back Projection)[1] and ART (Algebraic Reconstruction Techniques)[2].

In the Bayesian paradigm, the reconstructed image X is usually selected as

$$\hat{X} = \arg\max_X P(X|Y) = \arg\max_X P(Y|X)P(X), \quad (1)$$

where $P(X)$ is a prior distribution incorporating information about the expected structure in the image X, and $P(Y|X)$ models the degradation process of projections Y of the pixel intensities of the emission source (patient). In order to apply the Bayesian paradigm to the reconstruction of SPECT images we need therefore to specify the degradation and prior models and the process to obtain the MAP estimate.

The rest of the paper is organized as follow. In section 2 we define the degradation and image models used, and in section 3 we propose a method for finding the MAP estimate. The application of this method to synthetic and real images is described in section 4. Finally, section 5 concludes the paper.

2. DEGRADATION AND IMAGE MODELS

The degradation model for emission tomography can be specified as a product of independent Poisson distributions:

$$P(Y|X) = \prod_{s=1}^{M} \frac{(\sum_{i=1}^{N} A_{s,i}x_j)^{y_s} \exp\{-\sum_{t=1}^{N} A_{s,t}x_t\}}{y_s!}, \quad (2)$$

where M is the number of detectors, N the number of pixels in the image and A is the system matrix or discrete Radon transform (an element of this matrix $A_{s,i}$ represents the probability that an emitted photon from source pixel i reaches detector location s).

The prior model we use is a Compound Gauss-Markov Random Field (CGMRF) model. This model provides us with a means to control changes in the image using a hidden random field. A CGMRF model has two levels, an upper level which is the image to be restored and a lower or hidden level that it is a finite range random field to govern the transition between the sub-models. The use of an underlying random field, called the line process, was introduced by Geman and Geman [3] in the discrete case. Extensions

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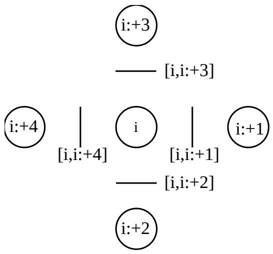

Fig. 1. Image and line sites

to the continuous case were presented by Jeng and Woods [4] and Chellapa et al. [5] (see also [6, 7] for medical image reconstruction).

The CGMRF model to be used is introduced from a simpler one, the Conditional Auto-Regression (CAR) model [8]. This prior model is defined by

$$P(X) \propto \exp\big\{-\frac{1}{2}\alpha\, X^t(I-\phi C)X\big\}, \qquad (3)$$

where the entries of C, $C_{i,j}$, are equal to one if pixels i and j are spatial neighbors and zero otherwise, α is a scaling parameter, and $|\phi| < 1/4$.

If we assume a ”toroidal edge correction”, from Eq. (3), we obtain

$$\begin{aligned} -\log P(X) &= \text{constant} + \frac{\alpha}{2}X^t(I-\phi C)X \\ &= \text{constant} + \frac{\alpha}{2}\sum_i \phi(x_i - x_{i:+1})^2 \\ &+ \frac{\alpha}{2}\sum_i \phi(x_i - x_{i:+2})^2 + \frac{\alpha}{2}\sum_i (1-4\,\phi)x_i^2, \end{aligned}$$

where i:+1, i:+2, i:+3, i:+4 are the four neighboring pixels around pixel i (see figure 1).

We now introduce the line process by rewriting this equation as

$$\begin{aligned} -\log P(X) &= \text{constant} \\ &+\frac{\alpha}{2}\sum_i \phi(x_i - x_{i:+1})^2(1-l_{[i,i:+1]}) \\ &+\frac{\alpha}{2}\sum_i \phi(x_i - x_{i:+2})^2(1-l_{[i,i:+2]}) \\ &+\frac{\alpha}{2}\sum_i \left[\beta l_{[i,i:+1]} + \beta l_{[i,i:+2]}\right] \\ &+\frac{\alpha}{2}\sum_i (1-4\,\phi)x_i^2, \end{aligned} \qquad (4)$$

where $l([i,j])$ takes the value of zero if pixels i and j are not separated by an active line and one otherwise. The parameter β is a scalar weight, which adjusts the introduction of the active line elements. For β very large the prior model becomes Gaussian. The process line acts as inhibitor or activator of the relation between two neighboring pixels depending on whether or not there exists an edge. Notice that the CAR model is obtained when $l[i,j] = 0, \forall i, j$.

3. MAP ESTIMATION

Let us now proceed to find $\hat{X}$, $\hat{L}$, the MAP estimates of X and L, that is

$$\hat{X}, \hat{L} = \arg\max_{X,L} P(X, L|Y). \qquad (5)$$

Since $P(X, L|Y)$ is nonlinear, it is difficult to find $\hat{X}$ and $\hat{L}$ by conventional methods. The method we propose for estimating the original image and the line process is stochastic for the line process and deterministic for the reconstruction, as described next.

In order to estimate the line process we simulate the corresponding conditional *a posteriori* density function. Let us denote by $P_T(l_{[i,j]}|L_{[i,j]}, X, Y)$ the conditional a posteriori density function for the line process $l_{[i,j]}$, given X, Y and the rest of L, $L_{[i,j]} = (l_{[s,t]} : \forall [s,t] \neq [i,j])$. To simulate this density function, we have

$$P_T(l_{[i,j]} = 0|L_{[i,j]}, X, Y) \propto \exp\left[-\frac{1}{T}\frac{\alpha\phi}{2}(x_i - x_j)^2\right] \quad (6)$$

$$P_T(l_{[i,j]} = 1|L_{[i,j]}, X, Y) \propto \exp\left[-\frac{1}{T}\frac{\alpha\beta}{2}\right], \quad (7)$$

where T is the temperature.

Given an estimate of the line process, L, and the observation, Y, we estimate the image X using the deterministic method in [9], extended for the use on tomographic images and modified to take into account only neighbors not separated by an active line element (see [10] for a discussion on iterative methods to reconstruct medical images). This method starts with the probability distribution

$$\begin{aligned} -\log P(X|L,Y) &= \text{constant} \\ &+\sum_{s=1}^{M}\Big(\sum_{i=1}^{N}(A_{s,i}x_j) - y_s \log\big[\sum_{t=1}^{N} A_{s,t}x_t\big]\Big) \\ &+\frac{\alpha}{2}\sum_i \phi(x_i - x_{i:+1})^2(1-l_{[i,i:+1]}) \\ &+\frac{\alpha}{2}\sum_i \phi(x_i - x_{i:+2})^2(1-l_{[i,i:+2]}) \\ &+\frac{\alpha}{2}\sum_i \left[\beta l_{[i,i:+1]} + \beta l_{[i,i:+2]}\right] \\ &+\frac{\alpha}{2}\sum_i (1-4\phi)x_i^2. \end{aligned} \qquad (8)$$

Differentiating $-\log P(X|L,Y)$ with respect to each pixel x_i, we obtain the following equation

$$1 + \alpha \left(\phi x_i \sum_{j \in \mathcal{N}_i} (1 - l_{[i,j]}) + (1 - 4\phi) x_i \right) = \sum_{s=1}^{M} \frac{y_s A_{s,i}}{\sum_{t=1}^{N} A_{s,t} x_t} + \alpha\phi \sum_{j \in \mathcal{N}_i} x_j (1 - l_{[i,j]}),$$

where $j \in \mathcal{N}_i$ denotes the neighboring pixels at distance one from i and we have assumed that $\sum_{s=1}^{M} A_{s,i} = 1$.

By adding $\alpha\phi x_i \sum_{j\in\mathcal{N}_i} l_{[i,j]}$ to both sides of the previous equation we obtain

$$1 + \alpha x_i = \sum_{s=1}^{M} \frac{y_s A_{s,i}}{\sum_{t=1}^{N} A_{s,t} x_t} + \alpha \left(\phi \sum_{j \in \mathcal{N}_i} x_j (1 - l_{[i,j]}) + \phi x_i \sum_{j \in \mathcal{N}_i} l_{[i,j]} \right)$$

or

$$\alpha^{-1} + x_i = \alpha^{-1} \sum_{s=1}^{M} \frac{y_s A_{s,i}}{\sum_{t=1}^{N} A_{s,t} x_t} + \phi \sum_{j \in \mathcal{N}_i} x_j (1 - l_{[i,j]}) + \phi x_i \sum_{j \in \mathcal{N}_i} l_{[i,j]}.$$

Multiplying both sides of the previous equation by x_i we obtain

$$x_i = \mu_i \left(\phi \sum_{j \in \mathcal{N}_i} x_j (1 - l_{[i,j]}) + \phi x_i \sum_{j \in \mathcal{N}_i} l_{[i,j]} \right) + (1 - \mu_i) x_i \sum_{s=1}^{M} \frac{y_s A_{s,i}}{\sum_{t=1}^{N} A_{s,t} x_t}, \quad (9)$$

with $\mu_i = x_i/(x_i + \alpha^{-1})$.

We can now use Eqs. (6), (7) and (9) in the following algorithm to find the MAP estimates of L and X:

Let $k = 1, 2, ...$, be the sequence of iterations in which the sites (lines or pixels) are visited for updating.

1. Set $k = 0$ and assign an initial configuration denoted as X_{-1}, L_{-1} and a initial temperature $T = 1$.
2. The evolution $\hat{L}_{k-1} \rightarrow \hat{L}_k$ of the line process is obtained simulating the probability functions defined in Eqs. (6) and (7).
3. The evolution $\hat{X}_{k-1} \rightarrow \hat{X}_k$ of the image is obtained using $\hat{X}_{k-1}$ in the right hand side of Eq. (9) and X_k in the left hand side of Eq. (9).
4. Set $k = k+1$. Decrease the temperature T according to an annealing scheme [4]. Go to step 2 until $k > N$, where N is a specified integer.

4. EXPERIMENTAL RESULTS

We compare the results using the proposed method with the reconstruction obtained by FBP and the CAR and GGMRF (Generalized Gauss Markov Random Field) [11] priors. For the two last models, the scaling parameters for the reconstructions were obtained using the estimation process described in [12].

The methods were tested on the 128x128 pixels synthetic image depicted in figure 2(a). A circular orbit with parallel hole collimator geometry is assumed. 128 detectors with 128 angles are simulated and the projection data Y are degraded with Poisson noise. The corresponding sinogram is shown in figure 2(b). We can observe that the method with a CAR prior penalizes in excess the edges (see figure 2(c)), while the method with the GGMRF prior (figure 2(d)) preserves the edges when the shape parameter is near 1 (for our experiments we used a shape parameter fixed to 1.1). The reconstruction with CGMRF (figure 2(e)) shows an improved reconstruction. Figure 2(f) shows the edges corresponding to figure 2(e). It is clear that the proposed method has captured the edges present in the image.

We also applied our reconstruction method to real images (see figure 3). The detector system was a Siemens Orbiter 66601, and the collimator was the parallel hole, medium energy, Siemens Anticamera 4445060 model. The gamma-camera described a circular orbit, at 5.625 degrees steps, for a total of 64 angles. The image corresponds to the inferior part of the liver (the right lobe and the center posterior hepatic zone, crossed there by blood vessels). The data provided by the detector system and its FBP reconstruction can be observed in figure 3(a) and (b), respectively. In figures 3(c) and 3(d) we show the reconstructions with CAR and GGMRF priors. The reconstruction with the proposed method and the corresponding line process are shown respectively in figures 3(e) and 3(f). The proposed method clearly increases the quality of the reconstruction since the areas with different levels of vascularization can be better distinguished.

5. CONCLUSIONS

In this paper we have presented a new method that can be used to reconstruct SPECT images. This method uses a prior model with a line process. The MAP estimation is performed by simulated annealing for the line process and a deterministic iterative scheme for the image. The experimental results show the validity of the method.

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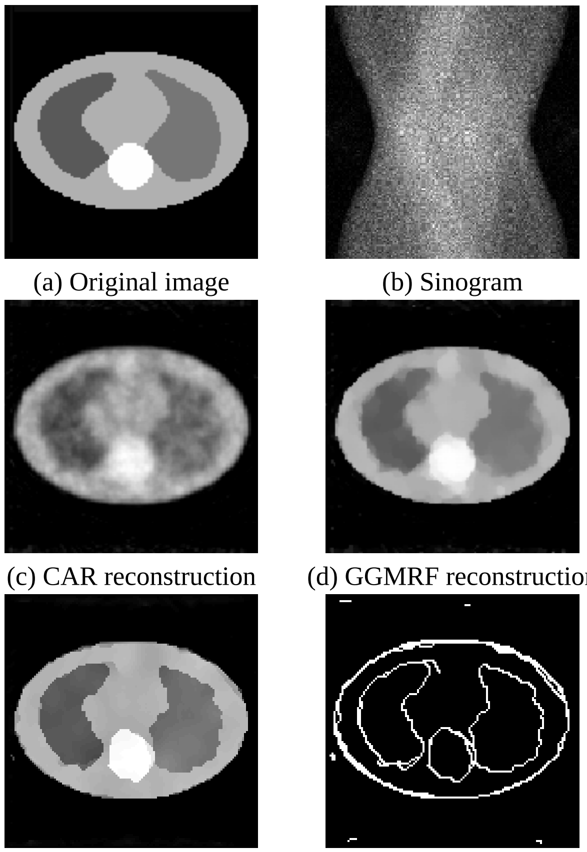
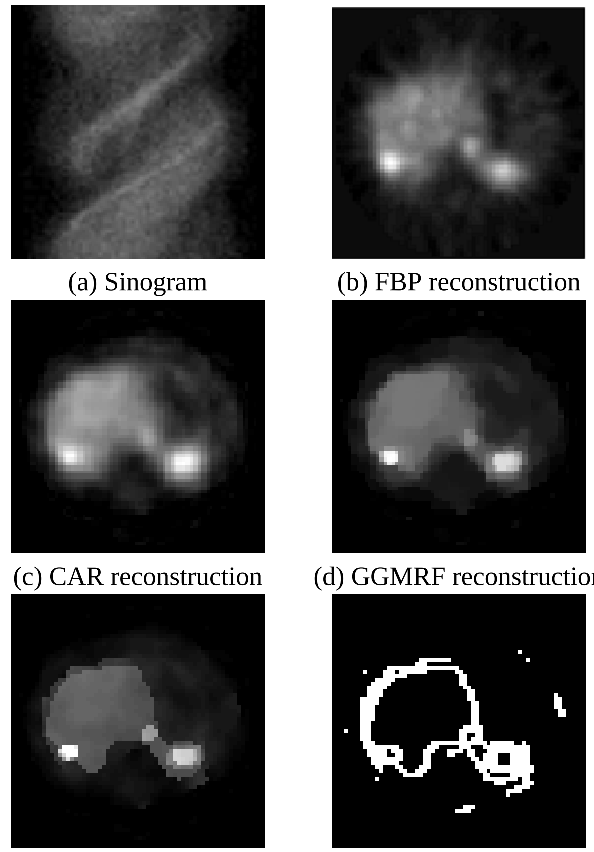


(a) Original image (b) Sinogram

(c) CAR reconstruction (d) GGMRF reconstruction

(e) CGMRF reconstruction (f) Line process

Fig. 2. Results with a synthetic image.



(a) Sinogram (b) FBP reconstruction

(c) CAR reconstruction (d) GGMRF reconstruction

(e) CGMRF reconstruction (f) Line process

Fig. 3. Results with a real image.

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