MULTICHANNEL IMAGE RESTORATION USING COMPOUND GAUSS-MARKOV RANDOM FIELDS

Rafael Molina, Javier Mateos *†

Depto. de Ciencias de la Computación e I. A.
Universidad de Granada
18071 Granada, Spain
{rms, jmd}@decsai.ugr.es

Aggelos K. Katsaggelos

Dept. of Electrical and Computer Eng.
Northwestern University
Evanston, IL
aggk@ece.nwu.edu

ABSTRACT

In this paper, a solution to the multichannel image restoration problem is provided using compound Gauss Markov random fields. For the single channel deblurring problem the convergence of the Simulated Annealing (SA) and Iterative Conditional Mode (ICM) algorithms has not been established. We propose two new iterative multichannel restoration algorithms which can be considered as extensions of the classical SA and ICM approaches and whose convergence is established. Experimental results with color images demonstrate the effectiveness of the proposed algorithms.

1. INTRODUCTION

Multichannel image processing differs from single channel (grayscale) image processing because of the redundancy and the complementary information within the channels. The processing is much more complicated due to the increased dimensionality of the problem and the need to extract and exchange information from and among all channels.

In the past some effort has been devoted to restore multichannel/color images (see [1, 2, 7] and the references therein) by enforcing the similarity between the intensity value of the pixels in the channels of the image. Although this approach gives good results if the wavelengths of the channels are close, in other cases it results in color bleeding. Here we will approach the problem from the point of view of similar regions.

In this paper we examine the use of compound Gauss Markov random fields (CGMRF) (see [3, 4, 5, 6]) to restore multichannel images. We propose two new iterative restoration algorithms which extend the application of the classical Simulated Annealing (SA) and Iterative Conditional Mode (ICM) approaches from single channel to multichannel images. It is mentioned here that the convergence of the SA and ICM algorithms has not been established for the single channel deblurring problem. The convergence of the proposed algorithms is established as an extension of [8] to multichannel images.

The paper is organized as follows. In section 2, notation, the proposed model for the image and line processes, and the noise model are introduced. Stochastic and deterministic relaxation approaches to obtain the maximum *a posteriori* (MAP) estimates are presented in section 3. Finally, the performance of the proposed methods is presented in section 4.

2. NOTATION AND MODEL

We will distinguish between f, the 'true' image which would be observed under ideal conditions (i.e., no noise and no distortions produced by blurring and instrumental effects), and g, the observed image. Let us assume that there are L channels, each of size $M \times N$, represented by $g^t = ({g^1}^t \; {g^2}^t \ldots {g^L}^t)$ and $f^t = ({f^1}^t \; {f^2}^t \ldots {f^L}^t)$, where each of the $M \times N$ vectors g^c, f^c, $c = 1, \ldots, L$, results from the lexicographic ordering of the two-dimensional signals in each channel. We will denote by f_i^c the intensity of the original channel c at the location of the pixel i on the lattice. The convention applies equally to the observed image g. The aim is then to reconstruct f from g.

When using a CGMRF, we need to define a *prior distribution*, a probability distribution over images f where we incorporate information on the expected structure within an image. In this prior distribution we also introduce a line process, l, that, intuitively, acts as an activator or inhibitor of the relation between two neighboring pixels depending on whether or not the pixels

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are separated by an edge. It is also necessary to specify $p(g|f,l)$, the probability distribution of observed images g if f were the 'true' image and l the line process, that models how the observed image has been obtained from the 'true' one. The Bayesian paradigm dictates that inference about the true f and l should be based on $p(f,l|g)$ given by

$$p(f,l|g) \propto p(g|f,l)p(f,l). \quad (1)$$

Maximization of (1) with respect to f and l yields

$$\hat{f},\hat{l} = \arg\max_{f,l} p(f,l|g), \quad (2)$$

the maximum *a posteriori* (MAP) estimator. Let us examine the degradation and prior models.

The degradation model we consider is given by

$$g = Hf + n, \quad (3)$$

where g, f and n represent the observed image, the original image and the noise, respectively. We assume, for the sake of simplicity, that the $[L(M\times N)]\times[L(M\times N)]$ multichannel degradation matrix H is equal to $H = \mathrm{diag}(H^1\ H^2\dots H^L)$, that is, no cross-channel degradation is present. However, the method we are presenting here can be extended to handle cross-channel degradations. We therefore have

$$p(g|f,l) \propto \exp\left\{-\sum_{c=1}^{L}\frac{1}{2{\sigma_{\mathrm{n}}^{c}}^{2}} \parallel g^c - H^c f^c \parallel^2\right\}, \quad (4)$$

where ${\sigma_{\mathrm{n}}^{c}}^{2}$ is the noise variance for the channel c.

The prior model used is

$$\begin{aligned} p(f,l) \propto \exp\Bigg\{&-\sum_{c=1}^{L}\frac{1}{2{\sigma_{\mathrm{w}}^{c}}^{2}}\sum_i \Big[(1-4\phi){f_i^c}^2 \\ &+\phi(f_i^c - f_{i:+1}^c)^2(1-l_{[i,i:+1]}^c)+\beta l_{[i,i:+1]}^c \\ &+\phi(f_i^c - f_{i:+2}^c)^2(1-l_{[i,i:+2]}^c)+\beta l_{[i,i:+2]}^c\Big] \\ &+\sum_{\substack{c,c'=1\\ c\neq c'}}^{L}\sum_i\frac{\epsilon}{4}\left[l_{[i,i:+1]}^c l_{[i,i:+1]}^{c'} + l_{[i,i:+2]}^c l_{[i,i:+2]}^{c'}\right]\Bigg\}, \end{aligned} \quad (5)$$

where we are assuming a 'toroidal edge correction', $i:+1$, $i:+2$, $i:+3$, $i:+4$ denote the four pixels around pixel i (if $i=(u,v)$ they correspond to $(u+1,v),(u,v+1),(u-1,v)$ and $(u,v-1)$, respectively) and the line process between pixels i and j, $l_{[i,j]}^c$, takes the value zero if pixels i and j in channel c, $c=1,\dots,L$, are not separated by an active line and one otherwise. We then penalize the introduction of an active line element in the position $[i,j]$ by the term $\beta l_{[i,j]}^c$. By introducing the term $\epsilon(l_{[i,i:+1]}^c l_{[i,i:+1]}^{c'} + l_{[i,i:+2]}^c l_{[i,i:+2]}^{c'})$ with $c,c'=1,\dots,L$, $c\neq c'$ and $\epsilon>0$, we increase the probability of a new active line element in the position $[i,j]$ if the other channels present a line in the same position. Notice that ϵ could be dependent on the channel we are considering. The introduction of these cross terms will help to recognize the same objects in all the channels even if they do not have similar intensity. The parameter ${\sigma_{\mathrm{w}}^{c}}^{2}$ measures the smoothness of the 'true' channel c of the image and ϕ is just less than 0.25.

It could be argued that by the introduction of these cross-channels terms for the line process we may end up creating non-existing regions in some channels (for instance, in a RGB representation, a 50% red, 50% blue object in a 50% blue background may create the same object in all the bands). However, since we are smoothing using the four pixels around the current one, the introduction of a line will only force smoothing with fewer than four neighbors. Furthermore, the introduction of these terms will allow to recognize a 50% red, 50% blue object in a 40% blue background helping to create the same object in the blue band.

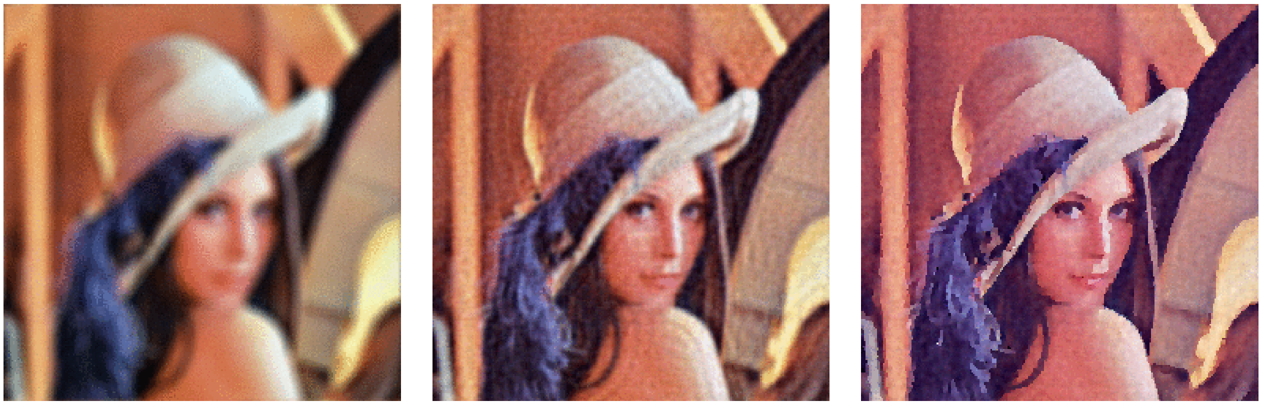
3. MAP ESTIMATION USING STOCHASTIC RELAXATION

Since $p(f,l|g)$ is nonlinear it is extremely difficult to find $\hat{f}$ and $\hat{l}$ by any conventional method. The method we use here is the modified Simulated Annealing technique proposed in [8], a relaxation technique to search for MAP estimates from degraded observations. It uses the distribution

$$\begin{aligned} p_T(f,l|g) = \frac{1}{Z_T}\exp\Bigg\{&-\frac{1}{T}\sum_{c=1}^{L}\Big[\frac{1}{2{\sigma_{\mathrm{n}}^{c}}^{2}}\parallel g^c - H^c f^c\parallel^2 \\ &+\frac{1}{2{\sigma_{\mathrm{w}}^{c}}^{2}}\sum_i\Big[(1-4\phi){f_i^c}^2 \\ &+\phi(f_i^c - f_{i:+1}^c)^2(1-l_{[i,i:+1]}^c)+\beta l_{[i,i:+1]}^c \\ &+\ \phi(f_i^c - f_{i:+2}^c)^2(1-l_{[i,i:+2]}^c)+\beta l_{[i,i:+2]}^c\Big]\Big] \\ &-\sum_{\substack{c,c'=1\\ c\neq c'}}^{L}\sum_i\frac{\epsilon}{4}\left[l_{[i,i:+1]}^c l_{[i,i:+1]}^{c'} + l_{[i,i:+2]}^c l_{[i,i:+2]}^{c'}\right]\Bigg\}, \end{aligned} \quad (6)$$

where T is the temperature and Z_T a normalization constant.

We shall need to simulate the conditional *a posteriori* density function for $l_{[i,j]}^c$, given the rest of l, f and g and the conditional *a posteriori* density function



(a) (b) (c)

Figure 1: (a) Degraded image, (b) restoration without line process and, (c) restoration with the proposed method.

for f_i^c given the rest of f, l and g. To simulate the line process conditional *a posteriori* density function, $p_T(l^c_{[i,j]}|l^c_{[k,l]} : \forall [k,l] \neq [i,j], l^{c'} : \forall c' \neq c, f, g)$, we have

$$p_T(l^c_{[i,j]} = 0|l^c_{[k,l]} : \forall [k,l] \neq [i,j], l^{c'} : \forall c' \neq c, f, g) \propto \exp\left[-\frac{1}{T}\frac{\phi}{2\sigma_{\mathrm{w}}^{c\,2}}(f_i^c - f_j^c)^2\right], \quad (7)$$

$$p_T(l^c_{[i,j]} = 1|l^c_{[k,l]} : \forall [k,l] \neq [i,j], l^{c'} : \forall c' \neq c, f, g) \propto \exp\left[-\frac{1}{T}\left(\frac{\beta}{2\sigma_{\mathrm{w}}^{c\,2}} - \epsilon \sum_{c'\neq c} l^{c'}_{[i,j]}\right)\right]. \quad (8)$$

For our Gaussian noise case, the conditional density function for f_i^c given the rest of f, l and g, $p_T(f_i^c|f_j^c : \forall j \neq i, f^{c'} : \forall c' \neq c, l, g)$, we will simulate is the Gaussian distribution defined by [8],

$$p_T(f_i^c|f_j^c : \forall j \neq i, f^{c'} : \forall c' \neq c, l, g) \sim \mathcal{N}\left(\mu_i^c, T\sigma_i^{c\,2}\right), \quad (9)$$

with mean

$$\mu_i^c = \lambda_i^c \left[\phi \sum_{j\,\mathrm{nhbr}\,i} (f_i^c - f_j^c(1 - l^c_{[i,j]})) + (1-4\phi) f_i^c\right] + (1-\lambda_i^c)\left[(H^{ct} g^c)_i - (H^{ct} H^c f^c)_i + f_i^c\right], \quad (10)$$

and

$$\sigma_i^{c\,2} = (1 - \omega_i^{c\,2}) \frac{\sigma_{\mathrm{w}}^{c\,2}\sigma_{\mathrm{n}}^{c\,2}}{nn_i^c\,\sigma_{\mathrm{n}}^{c\,2} + s\,\sigma_{\mathrm{w}}^{c\,2}}, \quad (11)$$

where s is the sum of the square of the coefficients defining the blur function, that is, $s = \sum_i h_i^{c\,2}$, $nn_i^c = \phi \sum_{j\,\mathrm{nhbr}\,i} (1 - l^c_{[i,j]}) + (1-4\phi)$, where the suffix '$j$ nhbr i' denotes the four neighbor pixels at distance one from pixel i, $\omega_i^c = (\sigma_{\mathrm{n}}^{c\,2}(1 - nn_i^c) + (1-s)\sigma_{\mathrm{w}}^{c\,2})/(\sigma_{\mathrm{n}}^{c\,2} + \sigma_{\mathrm{w}}^{c\,2})$, and $\lambda_i^c = \sigma_{\mathrm{n}}^{c\,2}/(\sigma_{\mathrm{n}}^{c\,2} + \sigma_{\mathrm{w}}^{c\,2})$. Note that, although according to $p_T(f_i^c|f_j^c : \forall j \neq i, f^{c'} : \forall c' \neq c, l, g)$, the value of a pixel, f_i^c, will depend on the value of the pixels in the other channels when we have cross channel blurring, the prior image model only introduces graylevel dependency within each channel.

Having defined the needed probability distributions, let us examine the modified SA and ICM algorithms. The convergence of the algorithms is established as an extension of [8], to restore multichannel images.

Algorithm 1 (MSA procedure)

1. *Set $t = 0$ and assign an initial configuration denoted as $f(-1)$, $l(-1)$ and initial temperature $T(0) = 1$.*

2. *The evolution $l(t-1) \rightarrow l(t)$ of the line process can be obtained by sampling the next point of the line process from the raster-scanning scheme based on the conditional probability mass function defined in (7) and (8) and keeping the rest of $l(t-1)$ unchanged.*

3. *Set $t = t + 1$. Go back to step 2 until a complete sweep of the field l is finished.*

4. *The evolution $f(t-1) \rightarrow f(t)$ of the image system can be obtained by sampling the next value of the whole multichannel image based on the conditional probability mass function given in (9)*

5. *Set $T(t) = C_T / \log(1 + k(t))$, where C_T is a constant and $k(t)$ is the sweep iteration number at time t.*

6. *Go to step 2 until $t > t_f$, where t_f is a specified integer.*

(a) (b) (c)

Figure 2: Line process obtained by the proposed algorithm for the , (a) red band, (b) green band and (c) blue band.

The modified ICM procedure is obtained by selecting in steps 2 and 4 of Algorithm 1 the mode of the corresponding transition probabilities.

The results in [8] guarantee that the MSA and ICM algorithms converge to a local MAP estimate, even in the presence of blurring.

4. EXPERIMENTAL RESULTS

The proposed modified SA method was tested on real color images represented in RGB format. Results are presented on the 'Lena' image, blurred with an out-of-focus point spread function, H^c, $c \in \{R, G, B\}$, with radius 5, and Gaussian noise with ${\sigma_{\rm n}^c}^2 = 4$, for all the color bands. We chose ${\sigma_{\rm w}^c}^2 = 50$, $\beta = 175$ and $\epsilon = 0.6$. Figure 1 shows the degraded image, the restoration obtained without line process, that is, $l_i^c = 0$, $\forall i, \forall c$, and the restoration with the proposed method. It is clear that the proposed method gives superior results by reducing the ringing around the edges which, indeed, are considerably sharper in the restoration with the line process. The line process obtained by the proposed algorithm for each band is shown in figure 2.

We also compared the proposed SA method with the method using a CGMRF version that processed each band independently, that is, $\epsilon = 0.0$ in (5). The proposed modified SA method better detected the objects in each band, the edges were clearer and had no color bleeding, which is a problem with the independent restoration algorithm.

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