Color Image Restoration Using Compound Gauss-Markov Random Fields

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ABSTRACT

In this work we extend the use of Compound Gauss Markov Random Fields to the restoration of color images. While most of the work in color image restoration is concentrated on enforcing similarity between the intensity values of the pixels in the image bands, we propose combining information by means of the line process. In order to find the multichannel restoration modified versions of ICM and SA are proposed. The methods are finally tested on real images.

1 Introduction

Color image processing differs from grayscale image processing because of the redundancy and the complementary information within the color bands. The processing is much more complicated due to the increased dimensionality of the problem and the need to extract and exchange information from and among all bands.

In the past some effort has been devoted to restore multichannel/color images (see [1],[2] and [7] and the references therein) by enforcing the similarity between the intensity value of the pixels in the bands of the image. Although this approach gives good results if the wavelengths of the bands are close, in other cases it results in color bleeding. Here we will approach the problem from the point of view of similar regions.

In this paper we examine the use of compound Gauss Markov random fields (CGMRF) (see [3, 4, 5, 6]) to restore color images. We propose two new iterative restoration algorithms which extend the classical SA and ICM approaches to color images. It is mentioned here that the convergence of the Simulated Annealing (SA) and Iterative Conditional Mode (ICM) algorithms has not been established for the single channel deblurring problem. The convergence of the proposed algorithm is established as an extension of [8] to multichannel images.

The paper is divided as follows. In section 2 we introduce the notation we use, the proposed model for the image and line processes and the noise model. Stochastic and deterministic relaxation approaches to obtain the maximum *a posteriori* (MAP) estimates are presented in section 3. Performance of the proposed methods is presented in section 4.

2 Notation and Model

We will distinguish between f, the 'true' image which would be observed under ideal conditions (i.e., no noise and no distortions produced by blurring and instrumental effects), and g, the observed image. If we assume, without loss of generality, an RGB format and each image size is $M \times N$, they can be represented by $g^t = \left(g^{R^t}\; g^{G^t}\; g^{B^t}\right)$ and $f^t = \left(f^{R^t}\; f^{G^t}\; f^{B^t}\right)$, where each of the $M \times N$ vectors g^c, f^c, $c \in \{R, G, B\}$, results from the lexicographic ordering of the two-dimensional signals in each channel. We will denote by f_i^c the intensity of the true color band c at the location of the pixel i on the lattice. The convention applies equally to the observed image g. The aim is then to reconstruct f from g.

When using a CGMRF, we need to define a *prior distribution*, a probability distribution over images f where we incorporate information on the expected structure within an image. In this prior distribution we also introduce a line process, l, that, intuitively, acts as an activator or inhibitor of the relation between two neighboring pixels depending on whether or not the pixels are separated by an edge. It is also necessary to specify $p(g|f, l)$, the probability distribution of observed images g if f were the 'true' image and l the line process, that models how the observed image has been obtained from the 'true' one. The Bayesian paradigm dictates that inference about the true f and l should be based on $p(f, l|g)$ given by

$$p(f, l|g) \propto p(g|f, l)p(f, l). \qquad (1)$$

Maximization of (1) with respect to f and l yields

$$\hat{f}, \hat{l} = \arg\max_{f,l} p(f, l \mid g), \qquad (2)$$

the maximum *a posteriori* (MAP) estimator. Let us examine the degradation and prior models.

The following degradation model is considered

$$g = Hf + n, \qquad (3)$$

where g, f and n represent the observed image, the original image and the noise respectively. We assume for simplicity that the $[3(M \times N)] \times [3(M \times N)]$ multichannel degradation matrix H is equal to $H = \mathrm{diag}(H^R \ H^G \ H^B)$, that is, no cross-channel degradation is present, in other words,

$$\begin{pmatrix} g^R \\ g^G \\ g^B \end{pmatrix} = \begin{pmatrix} H^R & 0 & 0 \\ 0 & H^G & 0 \\ 0 & 0 & H^B \end{pmatrix} \begin{pmatrix} f^R \\ f^G \\ f^B \end{pmatrix} + \begin{pmatrix} n^R \\ n^G \\ n^B \end{pmatrix},$$

however, the method we are presenting can be extended to handle cross-channel degradations. We therefore have

$$\begin{aligned} p(g|f,l) \quad \propto \quad & \exp\left[-\frac{1}{2{\sigma_{\mathrm{n}}^{R}}^2} \parallel g^R - H^R f^R \parallel^2 \right. \\ & -\frac{1}{2{\sigma_{\mathrm{n}}^{G}}^2} \parallel g^G - H^G f^G \parallel^2 \\ & \left. -\frac{1}{2{\sigma_{\mathrm{n}}^{B}}^2} \parallel g^B - H^B f^B \parallel^2 \right]. \end{aligned} \tag{4}$$

The prior model used is

$$\begin{aligned} p(f,l) \propto \exp & \left\{ - \sum_{c \in \{R,G,B\}} \right. \\ & \frac{1}{2{\sigma_{\mathrm{w}}^{c}}^2} \sum_i \Big[\phi(f_i^c - f_{i:+1}^c)^2 (1 - l_{[i,i:+1]}^c) \\ & + \beta l_{[i,i:+1]}^c + \phi(f_i^c - f_{i:+2}^c)^2 (1 - l_{[i,i:+2]}^c) \\ & + \beta l_{[i,i:+2]}^c + (1 - 4\phi) {f_i^c}^2 + \Big] \\ & + \frac{1}{4} \sum_{\substack{c,c' \in \{R,G,B\} \\ c \neq c'}} \sum_i \epsilon \Big[l_{[i,i:+1]}^c l_{[i,i:+1]}^{c'} \\ & \left. + l_{[i,i:+2]}^c l_{[i,i:+2]}^{c'} \Big] \right\}, \end{aligned} \tag{5}$$

where we are assuming a 'toroidal edge correction' and a toroidal correction on the color bands, $i:+1$, $i:+2$, $i:+3$, $i:+4$ denote the four pixels around pixel i (if $i = (u,v)$ they correspond to $(u+1,v)$, $(u,v+1)$, $(u-1,v)$ and $(u,v-1)$, respectively) and the line process between pixels i and j, $l_{[i,j]}^c$, takes the value zero if pixels i and j in band c, $c \in \{R,G,B\}$, are not separated by an active line and one otherwise (see Figure 1) . We then penalize the introduction of an active line element in the position $[i,j]$ by the term $\beta l_{[i,j]}^c$. By introducing the term $\epsilon(l_{[i,i:+1]}^c l_{[i,i:+1]}^{c'} + l_{[i,i:+2]}^c l_{[i,i:+2]}^{c'})$ with $c, c' \in \{R,G,B\}$, $c \neq c\prime$ and $\epsilon > 0$, we increase the probability of a new active line element in the position $[i,j]$ if the other bands present a line in the same position (see Figure 2). Notice that ϵ could be dependent on the bands we are considering. The introduction of these cross terms will help to recognize the same objects in all the bands even if they do not have similar intensity. The parameter ${\sigma_{\mathrm{w}}^{c}}^2$ measures the smoothness of the 'true' band c of the image and ϕ is just less than 0.25.

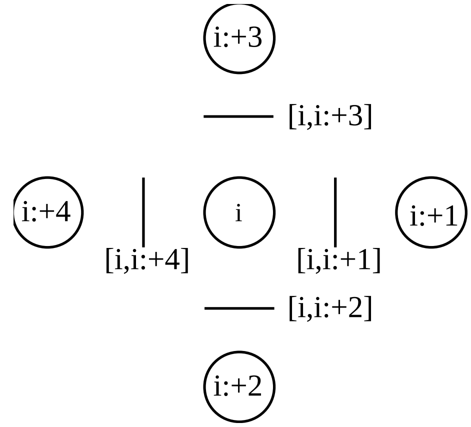

Figure 1: Image and line sites at each channel.

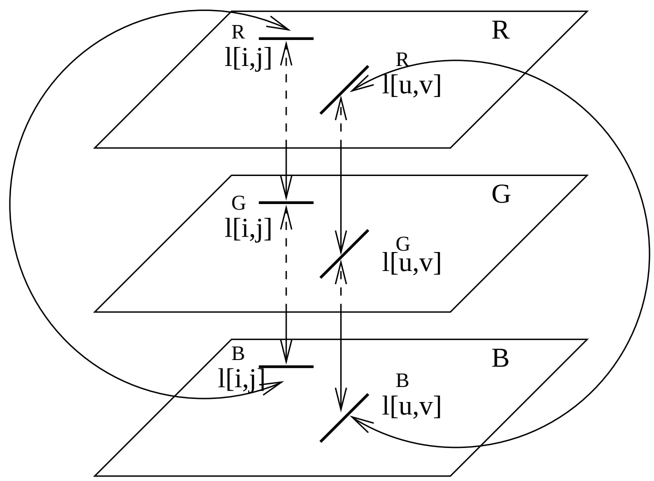

Figure 2: Interactions between line processes at different bands.

It could be argued that by the introduction of these cross-channels terms we may end up creating non-existing regions in some channels (for instance, a pure red object in a black background will help the creation of the same object in the green and blue bands). However, since we are smoothing using the four pixels around the current one, the introduction of a line will only force smoothing with fewer than four neighbors. Assuming that regions are more or less flat this should not create any problems.

3 MAP estimation using Stochastic Relaxation

Since $p(f,l \mid g)$ is nonlinear it is extremely difficult to find $\hat{f}$ and $\hat{l}$ by any conventional method. The method we use here is the modified simulated annealing technique proposed in [8], a relaxation technique to search for MAP estimates from degraded observations. It uses the distribution

$$p_T(f,l \mid g) = \frac{1}{Z_T} \exp\left\{ -\frac{1}{T} \sum_{c \in \{R,G,B\}} \right.$$

$$\left[\frac{1}{2{\sigma_{\rm n}^c}^2} \parallel g^c - H^c f^c \parallel^2 \right.$$
$$-\frac{1}{4}\sum_i \epsilon \left[l^c_{[i,i:+1]} l^{c+1}_{[i,i:+1]} + l^c_{[i,i:+2]} l^{c+1}_{[i,i:+2]}\right]$$
$$+\frac{1}{2{\sigma_{\rm w}^c}^2}\sum_i \left[\phi(f_i^c - f_{i:+1}^c)^2(1-l^c_{[i,i:+1]}) + \beta l^c_{[i,i:+1]}\right.$$
$$+\phi(f_i^c - f_{i:+2}^c)^2(1-l^c_{[i,i:+2]}) + \beta l^c_{[i,i:+2]}$$
$$\left.\left.\left. +(1-4\phi){f_i^c}^2\right]\right]\right\}, \qquad (6)$$

where T is the temperature and Z_T a normalization constant.

We shall need to simulate the conditional *a posteriori* density function for $l^c_{[i,j]}$, given the rest of l, f and g and the conditional *a posteriori* density function for f_i^c given the rest of f, l and g. To simulate the line process conditional *a posteriori* density function, $p_T(l^c_{[i,j]} \mid l^c_{[k,l]} : \forall[k,l] \neq [i,j], l^{c-1}, l^{c+1}, f, g)$, we have

$$p_T(l^c_{[i,j]} = 0 \mid l^c_{[k,l]} : \forall[k,l] \neq [i,j], l^{c-1}, l^{c+1}, f, g)$$
$$\propto \exp\left[-\frac{1}{T}\frac{\phi}{2{\sigma_{\rm w}^c}^2}(f_i^c - f_j^c)^2\right], \qquad (7)$$

and

$$p_T(l^c_{[i,j]} = 1 \mid l^c_{[k,l]} : \forall[k,l] \neq [i,j], l^{c-1}, l^{c+1}, f, g)$$
$$\propto \exp\left[-\frac{1}{2T}\left(\frac{\beta}{{\sigma_{\rm w}^c}^2} - \epsilon(l^{c-1}_{[i,j]} + l^{c+1}_{[i,j]})\right)\right]. \qquad (8)$$

The conditional *a posteriori* density function for f_i^c given the rest of f, l and g, $p_T(f_i^c \mid f_j^c : \forall j \neq i, f^{c-1}, f^{c+1}, l, g)$, as described in [8].

Notice that the modified ICM method defined in [8] can also extended to be used with multichannel images. The results in that paper guarantee its convergence to a local maximum.

4 Experimental Results

The proposed modified SA method was tested on real color images. Results are presented first on the 'Lena' image, blurred with an out-of-focus point spread function, H^c, $c \in \{R, G, B\}$, with radius 5, and Gaussian noise with ${\sigma_{\rm n}^c}^2 = 4$, for all the color bands. We chose ${\sigma_{\rm w}^c}^2 = 50$, $\beta = 175$ and $\epsilon = 0.6$. Figure $3(a)-(c)$ shows the degraded image, the restoration obtained without line process, that is, $l_i^c = 0$, $\forall i, \forall c$, and the restoration with the proposed method. It is clear that the proposed method gives superior results by reducing the ringing around the edges which, indeed, are considerably sharper in the restoration with the line process. We also compared the method with the method using a CGMRF version that processed each band independently, that is, $\epsilon = 0.0$ in Eq. (5) (see Figure $3d$). The proposed modified SA method better detected the objects in each band, the edges were clearer and had no color bleeding, which is a problem with the independent restoration algorithm.

Acknowledgement

This paper was partially supported by the "Comisión Nacional de Ciencia y Tecnología" under contract TIC97-0989.

The work of Javier Mateos was partially supported by the Spanish "Ministerio de Educación y Cultura".

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(a) (b) (c) (d)

Figure 3: (a) Degraded color Lena image, (b) restoration without line process and, (c) restoration with the proposed method. (d) restoration with line process in each band but without interactions between bands.